NOISE: Measurement and Generation

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Introduction

As anyone who has listened to a receiver suspects, everything in the universe generates noise. In communications, the goal is to maximize the desired signal in relation to the undesired noise we hear. In order to accomplish this goal, it would be helpful to understand where noise originates, how much our own receiver adds to the noise we hear, and how to minimize it.

It is difficult to improve something unless we are able to measure it. Measurement of noise in receivers does not seem to be clearly understood by many amateurs, so I will attempt to explain the concepts and clarify the techniques, and to describe the standard "measure of merit" for receiver noise performance: "noise figure." Most important, I will describe how to build your own noise generator for noise figure measurements.

A number of equations are included, but only a few need be used to perform noise figure measurements. The rest are included to as an aid to understanding, with, I hope, enough explanatory text for everyone.

Noise

The most pervasive source of noise is thermal noise, due to the motion of thermally agitated free electrons in a conductor. Since everything in the universe is at some temperature above absolute zero, every conductor must generate noise.

Every resistor (and all conductors have resistance) generates an *rms* noise voltage:

$$e = \sqrt{4kTRB}$$

where R is the resistance, T is the absolute temperature in degrees K, B is the bandwidth in Hertz, and k is Boltzmann's constant, 1.38×10^{-23} joules / K.

Converting to power, e^2/R , and adjusting for the Gaussian distribution of noise voltage, the noise power generated by the resistor is:

$$Pn = kTB \text{ (watts)}$$

which is independent of the resistance. Thus, all resistors at the same temperature generate the same noise power. The noise is white noise, meaning that the power density does not vary with frequency, but always has a power density of kT watts/Hz. More important is that the noise power is directly proportional to absolute temperature T, since k is a constant. At the nominal ambient temperature of 290 K, we can calculate this power; converted to dBm, we get the familiar -174 dBm/Hz. Just multiply by the bandwidth in Hertz to get the available noise power at ambient temperature. The choice of 290 K for ambient might seem a bit cool, since the equivalent 17° C or 62° F would be a rather cool room temperature, but 290 makes all the calculations come out to even numbers.

The *instantaneous noise voltage* has a Gaussian distribution around the *rms* value. The Gaussian distribution has no limit on the peak amplitude, so at any instant the noise voltage may have any value from -infinity to +infinity. For design purposes, we can use a value that will not be exceeded more than

0.01% of the time. This voltage is 4 times the *rms* value, or 12 dB higher, so our system must be able to handle peak powers 12 dB higher than the average noise power if we are to measure noise without errors.[1]

Signal to Noise Ratio (S/N)

Now that we know the noise power in a given bandwidth, we can easily calculate how much signal is required to achieve a desired signal to noise ratio, S/N. For SSB, perhaps 10 dB S/N is required for good communications; since ambient thermal noise in a 2.5 kHz bandwidth is -140 dBm, calculated as follows:

$$Pn = kTB = 1.38 \text{ x } 10^{-23} \times 290 \times 2500 = 1.0 \text{ x } 10^{-17} \text{ watts}$$

$$\text{dBm} = 10 \log (Pn \times 1000) \text{ [multiplying the power by 1000 to get milliwatts]}$$

The signal power must be 10 dB larger, so minimum signal level of -130 dBm is required for a 10 dB S/N. This represents the noise and signal power levels at the antenna. We are then faced with the task of amplifying the signal without degrading the signal to noise ratio.

Noise Temperature

Any amplifier will add additional noise. The input noise N_i per unit bandwidth is kT_g is amplified by gain G to produce an output noise of kT_gG. The additional noise, kT_n is added to produce a total noise output power N_o:

$$N_o = kT_gG + kT_n$$

To simplify future calculations, we pretend that the amplifier is noise-free but has an additional noise generating resistor of temperature T_e at the input, so that all sources of noise are inputs to the amplifier. Then the output noise is:

$$N_o = kG (T_g + T_e)$$

where T_e is the *Noise Temperature* of the excess noise contributed by the amplifier. The noise added by an amplifier is then kGT_e, which is the fictitious noise source at the input amplified by the amplifier gain.

Cascaded Amplifiers

If several amplifiers are cascaded, the output noise N_o of each becomes the input noise T_g to the next stage and we can create a large equation for the total. After removing the original input noise term, we are left with the added noise:

$$N_{added} = (k\, T_{e1}G_1G_2 \ldots G_N) + (kT_{e2}G_2 \ldots G_N) + \ldots + (kT_{eN}G_N)$$

Substituting in the total gain $G_T = (G_1G_2 \ldots G_N)$ results in the total excess noise:

$$T_{eT} = T_{e1} + T_{e2}/G_1 + T_{e2}/G_1G_2 + \ldots + T_{eN}/(G_1G_2 \ldots G_{N-1})$$

showing that the noise of each succeeding stage reduced by the gain of all preceding stages. Clearly, if the gain of the first stage, G_1, is large, then the noise contributions of the succeeding stages are not significant. This is why we concentrate our efforts on improving the first amplifier or preamplifier.

Noise Figure (NF)

The *noise figure* (NF) of an amplifier is the logarithm of the ratio (so we can express it in dB) of the total noise output of an amplifier with an input T_g of 290 K to the noise output of an equivalent noise-free amplifier. A more useful definition is to calculate it from the excess temperature T_e:

$$NF = 10 \log (1 + T_e / T_0) \text{ (dB) @ } T_0 = 290 \text{ K}$$

If the NF is known, then T_e may be calculated after converting the NF to a ratio, F:

$$T_e = (F - 1)\, T_0$$

Typically, T_e is specified for very low noise amplifiers, where the NF would be fraction of a dB, and NF is used when it seems a more manageable number than thousands of K.

Losses

We know that any loss or attenuation in a system reduces the signal level. If attenuation also reduced the noise level, then we could suppress thermal noise by adding attenuation. We know intuitively that this can't be true. The answer is that the attenuator or any lossy element has a physical temperature, T_x , which contributes noise to the system while the input noise is being attenuated. The output noise after a loss L (ratio) is:

$$T_g' = T_g / L + [(L-1)/L]\, T_x$$

If the source temperature T_g is higher than the attenuator temperature T_x , then the noise contribution is the familiar result found by simply adding the loss in dB to the NF. However, for low source temperatures the degradation can be much more dramatic. If we do a calculation for the affect of 1 dB of loss (L = 1.26) on a T_g of 25 K:

$$T_g' = 25/1.26 + (0.26/1.26) \times 290 = 80 \text{ K}$$

The resultant T_g' is 80 K, a 5 dB increase in noise power (or 5 dB degradation of signal to noise ratio). Since noise power = kT and k is a constant, the increase is the ratio of the two temperatures 80/25, or in dB, $10\log(80/25) = 5$ dB.

Antenna Temperature

How can we have a source temperature much lower than ambient? If an antenna, assumed to be lossless, is receiving signals from space, rather than the warm earth, then the background noise is much lower. The background temperature of the universe has been measured as about 3.2 K. An empirical number for a 10 GHz antenna pointing into clear sky is about 6 K, since we must always look through attenuation and temperature of the atmosphere.[2] The figure will vary with frequency, but a good EME antenna might have a T_g of around 20 K at UHF and higher frequencies.

A couple of examples of actual antenna might bring all of this together.[3]

1 A 30 inch conventional dish at 10 GHz, with measured gain of 36.4 dBi and efficiency of 64%. The estimated spillover efficiency is 87% for a 10 dB illumination taper. With the dish pointing at a high elevation as shown in Figure 1, perhaps half of the spillover is illuminating earth at 290 K, which adds an estimated 19 K to the 6 K of sky noise, for a total of 25 K. In a 500 Hz bandwidth, the noise output is -157.6 dBm.

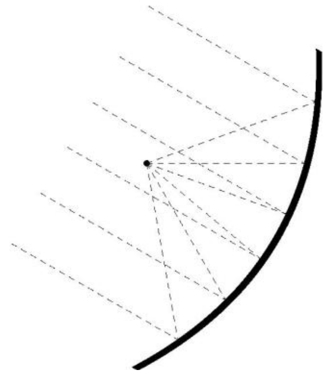


Figure 1. Parabolic Dish Antenna Aimed at Satellite

2 An 18 inch DSS offset-fed dish at 10 GHz, with measured gain of 32.0 dB and efficiency of 63%. The spillover efficiency should be comparable, but with the offset dish pointing at a high elevation as shown in Figure 2, far less of the spillover is illuminating warm earth. If we estimate 20%, then 8 K is added to the 6 K of sky noise, for a total of 14 K. In a 500 Hz bandwidth, the noise output is -160 dBm.

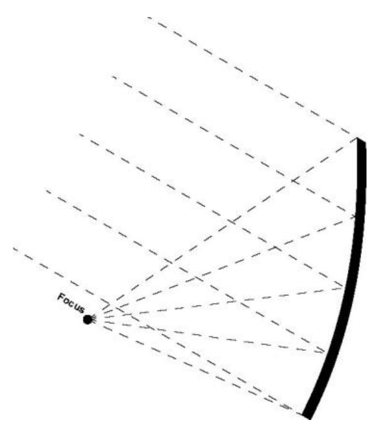


Figure 2. Offset Parabolic Dish Antenna Aimed at Satellite

The larger conventional dish has 2.4 dB higher noise output, but 4.4 dB higher gain, so it should have 2.0 dB better signal to noise ratio than the smaller offset dish when both are pointing at high elevations.

However, the while the offset dish is easy to feed with low loss, it is convenient to feed the conventional dish through a cable with 1 dB of loss. Referring back to our loss example above, the noise temperature after this cable loss is 80 K. In a 500 Hz bandwidth, the noise output is now -152.6 dBm, 7.4 dB worse than the offset dish. The convenience of the cable reduces the signal to noise ratio by 5 dB, making the larger conventional dish 3 dB worse than the smaller offset dish. Is it any wonder that the DSS dishes sprouting on rooftops everywhere are offset-fed?

If the dishes are pointed on the horizon for terrestrial operation, then the situation is much different. At least half of each antenna pattern is illuminating warm earth, so we should expect the noise temperature to be at least half of 290 K, or about 150 K. Adding 1 dB of loss increases the noise temperature to 179 K, a

1 dB increase. At the higher noise temperatures, losses do not have a dramatic effect on signal to noise ratio. In practice, the antenna temperature on the horizon may be even higher, since the upper half of the pattern must take a much longer path through the warm atmosphere, which adds noise just like any other loss.

Image Response

Most receiving systems use at least one frequency converting mixer which has two responses, the desired frequency and an image frequency on the other side of the local oscillator. If the image response is not filtered out, it will add additional noise to the mixer output. Since most preamps are broadband enough to have significant gain (and thus, noise output) at the image frequency, the filter must be placed between the preamp and the mixer. The total NF including image response is calculated:

$$NF = 10\log[(1 + T_e/T_0)(1 + G_{image}/G_{desired})]$$

assuming equal noise bandwidth for desired and image responses. Without any filtering, $G_{image} = G_{desired}$ so $G_{image}/G_{desired} = 1$, doubling the noise figure which is the same as adding 3 dB. Thus, without any image rejection, the overall noise figure is at least 3 dB <u>regardless of the NF of the preamp.</u> For the image to add less than 0.1 dB to the overall NF, a quick calculation shows that the gain at the image frequency must be at least 16 dB lower than at the operating frequency.

Noise Figure Measurement

So far we have discussed the sources of noise, and a figure of for evaluating a receiving system's response to noise. How can we measure an actual receiver?

The noise figure of a receiver is determined by measuring its output with two different noise levels, T_{hot} and T_{cold}, applied at the input. The ratio of the two output levels is referred to as the "Y-factor". Usually, the ratio is determined from the difference in dB between the two output levels, Y_{db}:

$$Y_{(ratio)} = \log^{-1}(Y_{db}/10)$$

Then the receiver T_e may be calculated using $Y_{(ratio)}$:

$$T_e = (T_{hot} - YT_{cold}) / (Y-1)$$

and converted to noise figure:

$$NF = 10\log(1 + T_e/T_0) \text{ (dB) where } T_0 = 290 \text{ K}$$

The two different noise levels may be generated separately, for instance by connecting resistors at two different temperatures. Alternatively, we could use a device that can generate a calibrated amount of noise when it is turned on. When such a device is turned off, it still generates noise from its internal resistance at T_{cold}, the ambient temperature (290 K); usually this resistance is 50 ohms, to properly terminate the transmission line which connects it to the receiver. When the noise generator is turned on, it produces excess noise equivalent to a resistor at some higher temperature at T_{hot}.

The noise produced by a noise source may be specified as the *Excess Noise Ratio* (ENR_{dB}), the dB difference between the cold and the equivalent hot temperature, or as the equivalent temperature of the excess noise, T_{ex}, which is used in place of T_{hot} in the previous equation. If the ENR is specified, then the calculation is:

$$NF_{dB} = ENR_{dB} - 10\,log(Y_{(ratio)} - 1)$$

The terms T_{ex} and ENR are used rather loosely; assume that a noise source specified in dB refers to ENR_{dB}, while a specification in degrees or K refers to T_{ex}.

An automatic noise figure meter, sometimes called a PANFI (for Precision Automatic Noise Figure Meter), turns the noise source on and off at a rate of about 400 Hz and performs the above calculation electronically.[4] A wide bandwidth is required to detect enough noise to operate at this rate; a manual measurement using a narrowband communications receiver would require the switching rate to be less than one Hz, with some kind of electronic integration to properly average the Gaussian noise.

Noise figure meters seem to be fairly common surplus items. The one most recently in production, the HP or Agilent 8970, measures both noise figure and gain, but commands a stiff price. Current instruments that measure noise figure include many other features at a price well beyond amateur budgets.

AIL (later AILTECH or Eaton) made several models; the model 2075 measures both NF and gain, while other models are NF only. The Model 75 (a whole series whose model numbers start with 75) shows up frequently for anywhere from $7 to $400, typically $25 to $50, and performs well. Every VHFer I know has one, with most of them waiting for a noise source to be usable. Earlier tube models, like the AIL 74 and the HP 340 and 342, have problems with drift and heat, but they can also do the job.

Another alternative is to build a noise figure meter.[5]

Using the Noise Figure Meter

I'll describe the basic procedure using the Model 75; others are similar, but the more complex instruments will require studying the instruction manual.

Input to almost all noise figure meters is at 30 MHz, so a frequency converter is required (some instruments have internal frequency converters; except for the HP 8970, I'd avoid using this feature). Most ham converters with a 28 MHz IF work fine, unless the preamp being measured is so narrowband that a MHz or two changes the NF. The input is fairly broadband, so LO leakage or any other stray signals can upset the measurement – this has been a source of frustration for many users. There are two solutions: a filter (30 MHz low-pass TVI filters are often sufficient) or a tuned amplifier at 30 MHz. Since a fair amount of gain is required in front of the noise figure meter, an amplifier is usually required anyway.

A noise source (which we will discuss in detail later) is connected to the rear of the instrument: a BNC connector marked "DIODE GATE" provides +28 volts for a solid-state noise source, and high voltage leads for a gas tube noise source are also available on many versions. The noise figure meter switches the noise source on and off. The noise output coax connector of the noise source is connected to the receiver input.

The Model 75 has four function pushbuttons: *OFF, ON, AUTO,* and *CAL.* The *OFF* and *ON* positions are for manual measurements: *OFF* displays the detector output with the noise source turned off, and *ON* displays the detector output with the noise source turned on. If all is working, there should be more output in the *ON* position, and a step attenuator in the IF line may be used to determine the change in output, or Y-factor, to sanity-check our results. The knob marked "GAIN" is used to get the meter reading to a desirable part of the scale in the *OFF* and *ON* positions only; it has no effect on automatic measurements.

The *AUTO* position causes the instrument to turn the noise source on and off at about a 400 Hz rate and to calculate the NF from the detected change in noise. The Model 75 has a large green light near the meter which indicates that the input level is high enough for proper operation – add gain until the light comes on. Then the meter should indicate a noise figure, but not a meaningful one, since we must first set the ENR_{dB} using the *CAL* position. The lower scale on the meter is marked for from 14.5 to 16.5 dB of ENR; adjust the "CAL ADJ" knob until the reading in the *CAL* position matches the ENR of the noise source.

If the ENR of your noise source is outside the marked range, read the section below on homebrew noise sources.

Now that we have calibrated the meter for the ENR of the noise source, we may read the noise figure directly in the *AUTO* position. Before we believe it, a few sanity checks are in order:

1 Manual Y-factor measurement and calculation.
2 Insert a known attenuator between the noise source and preamp. The NF should increase by exactly the attenuation added.
3 Measure something with a known noise figure (known means measured elsewhere; a manufacturers claim is not necessarily enough).

Finally, too much gain in the system may also cause trouble, if the total noise power exceeds the level that an amplifier stage can handle without gain compression. Gain compression will be greater in the on state, so the detected Y-factor will be reduced, resulting in erroneously high indicated NF. The Gaussian distribution of the noise means that an amplifier must be able to handle 12 dB more than the average noise level without compression. One case where this is a problem is with a microwave transverter to a VHF or UHF IF followed by another converter to the 30 MHz noise figure meter, for too much total gain. I always place a step attenuator between the transverter and the converter which adjust until I can both add and subtract attenuation without changing the indicated noise figure.

One final precaution: noise figure meters have a very slow time constant, as long as 10 seconds for some of the older models, to smooth out the random nature of noise. If you are using the noise figure meter to "tweak" a receiver, *tune very slowly!*

Sky Noise Measurement

Another way to measure noise figure at microwave frequencies is by measurement of sky noise and ground noise.[3,6] Sky noise is very low, around 6 K at 10 GHz, for instance, and ground noise is due to the ground temperature, around 290 K, so the difference is nearly 290 K. At microwave frequencies we can use a manageable antenna that is sharp enough that almost no ground noise is received, even in sidelobes, when the antenna is pointed at a high elevation. A long horn would be a good antenna choice.

The antenna is pointed alternately at clear sky overhead, away from the sun or any obstruction, and at the ground. The difference in noise output is the Y-factor; since we know both noise temperatures, the receiver noise temperature is calculated using the $Y_{(ratio)}$:

$$T_e = (T_{hot} - YT_{cold}) / (Y-1)$$

The latest version of my microwave antenna program, HDL_ANT, will make this calculation.[3] Since the measured Y-factor will be relatively small, this measurement will only be accurate for relatively low noise figures. On the other hand, low noise figures are the most difficult to measure accurately using other techniques.

A system for measuring sun noise was described by G3WDG which also works well for measuring noise figure from sky noise.[7] He built a 144 MHz amplifier with moderate bandwidth using MMICs and helical filters which amplifies the transverter output to drive a surplus RF power meter. The newer solid-state power meters are stable enough to detect and display small changes in noise level, and the response is slow enough to smooth out flicker. Since my 10 GHz system has an IF output at 432 MHz, duplicating G3WDG's amplifier would not work. In the junk box I found some surplus broadband amplifiers and a couple of interdigital filters, and combined these to provide high gain with a few MHz bandwidth, arranged as shown in Figure 3. I found that roughly 60 dB of gain after the transverter was required to get a reasonable level on the power meter, while the G3WDG system has somewhat narrower bandwidth so more gain is required.

Several precautions are necessary:

1. Peak noise power must not exceed the level that any amplifier stage can handle without gain compression. Amplifiers with broadband noise output suffer gain compression at levels lower than found with signals, so be sure the amplifier compression point is at least 12 dB higher than the indicated average noise power.
2. Make sure no stray signals appear within the filter passband.
3. Foliage and other obstructions add thermal noise, which obscure the cold sky reading.
4. Low noise amplifiers are typically very sensitive to input mismatch, so the antenna must present a low VSWR to the preamp.

A noise figure meter could also be used as the indicator for the sky noise measurement, but a calibrated attenuator would be needed to determine the Y-factor. Using different equipment gives us an independent check of noise figure, so that we may have more confidence in our measurements.

Another choice for the indicator would be one of the software-defined radios. Those from Flex Radio Systems have accurate signal power readout, far better than a typical S-meter, and adjustable bandwidth and averaging time. Averaging for several seconds is needed to make the signal level stable with random noise.

W2IMU suggested that the same technique could be used for a large dish at lower frequencies.[8] With the dish pointing at clear sky, the feed horn is pointing at the reflector which shields it from the ground noise so it only sees the sky noise. If the feed horn is then removed and pointed at the ground, it will then see the ground noise.

Noise figure meters are convenient, but if you don't have one, the equipment for measuring sun and sky noise could also be used indoors with a noise source. The only complication is that the Y-factor could be much larger, pushing the limits of amplifier and power meter dynamic range

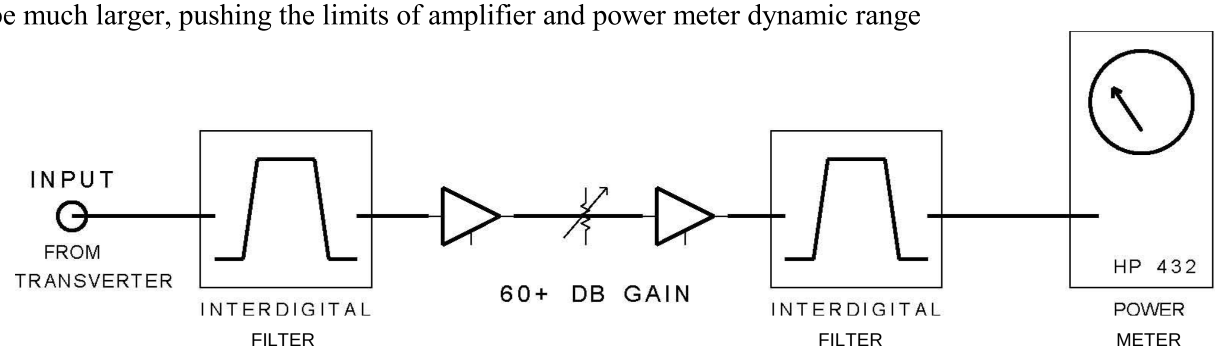


Figure 3. Indicator for Sun Noise

Noise Sources

The simplest noise source is simply a heated resistor – if we know the temperature of the resistor, we can calculate exactly how much noise it is generating. If we then change the temperature, the noise output will change by a known amount. This would work if we could find a resistor with good RF properties whose value does not change with temperature, an unlikely combination. There are commercial units, called Hot-Cold Noise Sources, with two calibrated resistors at different temperatures with low VSWR.

Typically, one resistor is cooled by liquid nitrogen to 77.3 K (the boiling point of nitrogen), while the other is heated by boiling water to 100°C, or 373.2 K. The preamp is connected to first one resistor, then the other; the difference in noise in noise output is the Y-factor. Using the $Y_{(ratio)}$, the preamp noise temperature is calculated:

$$T_e = (T_{hot} - YT_{cold}) / (Y-1)$$

Since the boiling point of pure liquids is accurately known, this type of noise generator can provide very accurate measurements. However, they are inconvenient to use, since the receiver must be connected directly to alternate resistors (the loss in an RF switch would significantly reduce the noise output and accuracy). Also, few amateurs have a convenient source of liquid nitrogen. The following three types of noise sources are commonly available and convenient to use.

Temperature-limited vacuum tube diode

The noise output is controlled by the diode current, but is only accurate up to around 300 MHz due to limitations of the vacuum tube. These units generate around 5 dB of excess noise.

Gas tube sources

The noise is generated by an ionized gas in the tube, similar to a fluorescent light – homebrew units have been built using small fluorescent tubes. The noise tubes use a pure gas, typically argon, to control the noise level. These units typically generate about 15 dB of excess noise.

Coaxial gas tube sources work up to around 2.5 GHz, and waveguide units to much higher frequencies. One problem using these is that a high voltage pulse is used to start the ionization (like the starter in a fluorescent light) which is coupled to the output in the coaxial units and is large enough to damage low-noise transistors. Since a noise figure meter turns the noise source on and off continuously, pulses are generated at the same rate.

Since waveguide acts as a high-pass filter, the starting pulses are not propagated to the output, so waveguide gas-tube noise sources are safe to use, though bulky and inconvenient. However, they could be used to calibrate a solid-state noise source.

Another problem with all gas tubes is that the VSWR of the noise source changes between the on and off states. If the source VSWR changes the noise figure of an amplifier, as is almost always the case, then the accuracy of the measurement is reduced.

Solid-state noise sources

Reverse breakdown of a silicon diode PN junction in causes an avalanche of current in the junction which would rise to destructively high levels if not limited by an external resistance. Since current is “electrons in motion,” a large amount of noise is generated. If the current density of the diode is constant, then the average noise output should also be constant; the instantaneous current is still random with a Gaussian distribution, so the generated noise is identical to thermal noise at a high temperature. Commercial units use special diodes designed for avalanche operation with very small capacitance for high frequency operation, but it is possible to make a very good noise source using the emitter-base junction of a small microwave transistor.

Typical noise output from an avalanche noise diode is 25 dB or more, so the output must be reduced to a usable level, frequently 15 dB of excess noise to be compatible with gas tubes or 5 dB of excess noise for more modern equipment. If the noise level is reduced by a good RF attenuator of 10 dB or more, then the source VSWR (seen by the receiver) is dominated by the attenuator, since the minimum return loss is twice the attenuation. Thus, the change in VSWR as the noise diode is turned on and off is minuscule. Commercial noise sources consist of a noise diode assembly and a selected coaxial attenuator permanently joined in a metal housing, calibrated as a single unit.

Homebrew Noise Sources

There are three components of a noise source: a noise generator, an attenuator, and the calibration data of ENR at each frequency. The most critical one is the attenuator; it is very important that the noise source present a very low VSWR to the preamp or whatever is being measured, since low-noise amplifiers are sensitive to input impedance, and even more important that the VSWR does not change when the noise source is turned on and off, since a change causes error in the measurement. Because an attenuator provides twice as many dB of isolation as loss (reflections pass through a second time), 10 dB or more of attenuation will reduce any change in VSWR to a very small value.

Commercial solid-state noise sources occasionally appear in surplus sources, usually at high prices but occasionally very cheap if no one knows what it is. I have found two of the latter, and one of them works! It produces about 25 dB of excess noise, which is too much to be usable. I went through my box of hamfest attenuators and found one which has excellent VSWR up to 10 GHz and 13 dB of attenuation. Mated with the noise source, the combination produces about 12 dB of excess noise — a very usable amount.

Finally, I calibrated it against a calibrated noise source for all ham bands between 50 MHz and 10 GHz; not exactly NTIS traceable, but pretty good for amateur work.

While noise sources are hard to locate, noise figure meters are frequent finds. If we could come up with some noise sources, all the VHFers who have one gathering dust could be measuring and optimizing their noise figure.

Several articles have described construction of homebrew noise sources, which work well at VHF and UHF, but not as well at 10 GHz.[9,10,11] All of them have the diode in a shunt configuration, with one end of the diode grounded. When I disassembled my defective commercial noise source (even the attenuator was bad), I found a bare chip diode in a series configuration – diode current flows into the output attenuator. Obviously I could not repair a chip diode, but I could try the series diode configuration. I found the smallest packaged microwave transistor available, some small chip resistors and capacitors, and soldered them directly on the gold-plated flange of an SMA connector with zero lead length, as shown in the photograph, Figure 4. We've all soldered components directly together in "dead-bug" construction; this is more like "fly-speck" construction.

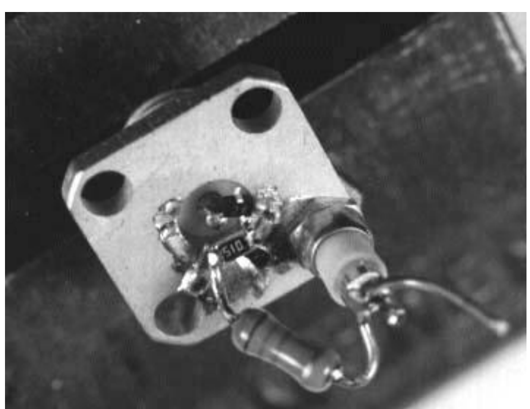


Figure 4

The schematic is shown in Figure 5, and it works at 10 GHz! I built several versions to evaluate reproducibility, and measured them at several ham bands from 30 MHz to 10 GHz, with results shown in Figure 6. All units were measured with the same 14 dB attenuator, so the diode noise generator output is 14 dB higher.

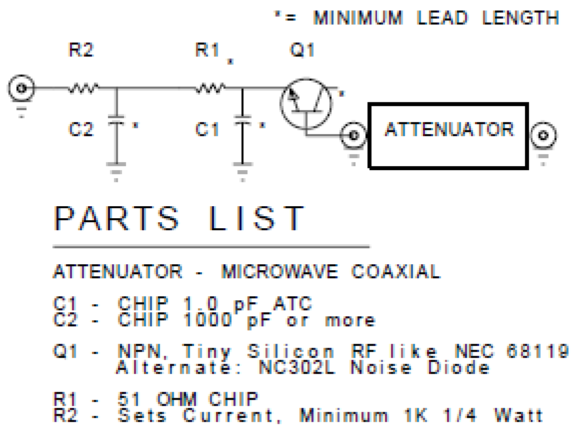


Figure 5. Noise Source Schematic

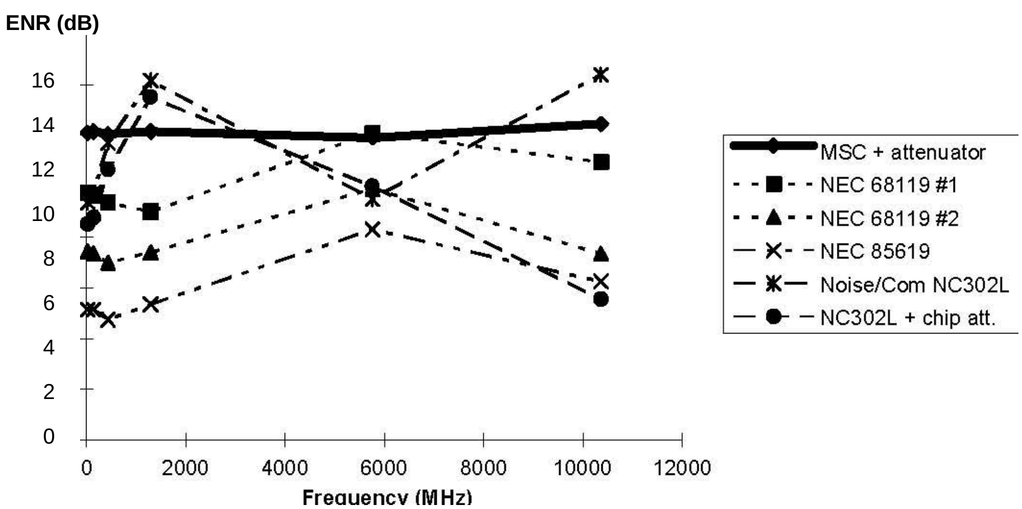


Figure 6. Noise Source ENR

(Later I found that the MIT Radiation Laboratory had described a noise source with a series diode 50 years ago, so we aren't giving away anyone's trade secrets.[12])

I then remembered that I had a commercial noise diode, a Noise/Com NC302L, which was used in a noise source described in *QST*, with the diode in the shunt configuration.[11] The diode is rated as working to 3 GHz, so, in the amateur tradition, I wanted to see if I could push it higher, using the series configuration. Since I didn't expect to reach 10 GHz, I increased the value of the bypass capacitor, but otherwise, it looks like the units in Figure 5. When I measured this unit, it not only worked at 10 GHz, but had more excess noise output than at lower frequencies, probably due to an unexpected resonance. The performance is shown in Figure 6 along with the other units.

Also shown in Figure 6 is the output of my pseudo-commercial noise source; even with the external attenuator, the excess noise output is pretty flat with frequency. Commercial units are typically specified at + or - 0.5 dB flatness. In Figure 6, none of the homebrew ones are that flat, but there is no need for it; as long as we know the excess noise output for a particular ham band, it is perfectly usable for that band.

All the above noise sources relied on a coaxial microwave attenuator to control the VSWR of the noise source. Attenuators are fairly frequent hamfest finds, but ones that themselves have good VSWR to 10 GHz are less common, and it's hard to tell how good they are without test equipment. An alternative might be to build an attenuator from small chip resistors. I used my *PAD.EXE* program to review possible resistor values, and found that I could make a 15.3 dB π attenuator using only 140 ohm resistors if the shunt legs were formed by two resistors in parallel, a good idea to reduce stray inductance.[13] I ordered some "0402" size (truly tiny) chip resistors from DigiKey, more NC302L diodes from Noise/Com, and built the noise source shown in Figure 7 on a bit of Teflon PC board, cutting out the 50 ohm transmission line with an X-Acto knife. The schematic of the complete noise source is shown in Figure 8.

The chip resistor attenuator works nearly as well as an expensive coaxial one. The measured VSWR of two noise sources, one with the chip attenuator and the other with a coaxial attenuator, is shown in Figure 9. Curves are shown in both the OFF and ON states, showing how little the VSWR changes. The VSWR of the chip attenuator unit is 1.42 at 10 GHz, slightly over the 1.35 maximum specified for commercial noise sources, but still fine for amateur use.

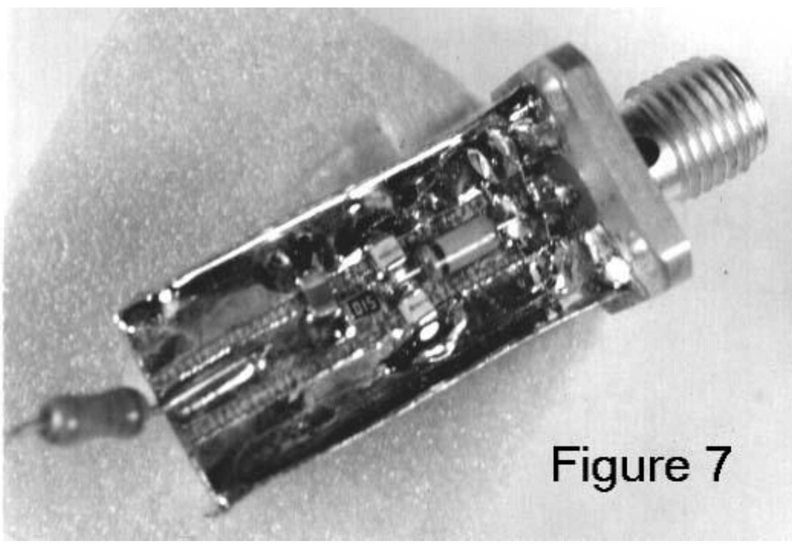
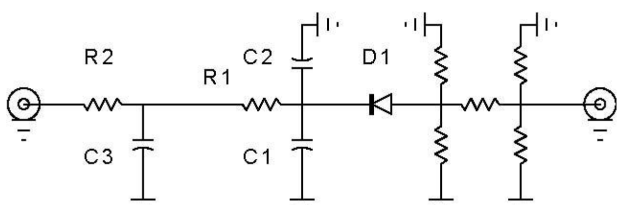


Figure 7



PARTS LIST

C1 -CHIP 1.0 pF ATCC2 -CHIP 8.2 pF ATCC2 -CHIP 1000 pF or more

D1 -NC302L Noise Diode

R1 -51 OHM CHIP R2 -Sets Current, Minimum 1K 1/4 Watt R3-7 -140 OHM CHIP, 0402 SIZE

Figure 8. Noise Source with Chip Attenuator

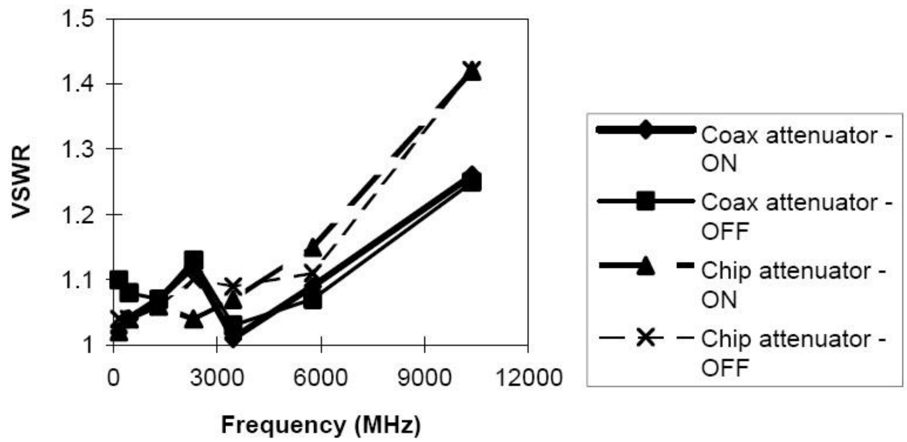


Figure 9. VSWR of Homebrew Noise Sources

Noise Source Alignment

The only alignment requirement for a solid-state noise source is to set the diode current; the current is always set at the highest frequency of interest. A noise figure meter must be set up with converters, etc., for the highest frequency at which the noise source might be used, and set to display the detector output (*OFF* position on a Model 75). Then voltage from a variable DC power supply is applied to the noise diode through the 1k current-limiting resistor. The detector output should increase as the voltage (diode current) increases, reach a peak, then decrease slightly. The optimum current is the one that produces peak output at the highest frequency (I set mine at 10 GHz). Then additional resistance must be added in series with the current-limiting resistor so that the peak output occurs with 28 volts applied, so that the noise source may be driven by the noise figure meter. Once the proper resistor is determined and added, the DC end of the noise source is connected to the diode output of the noise figure meter, and the meter function set to *ON*. This should produce the same detector output as the power supply. Then the meter function is set to *AUTO*, and the meter should produce some noise figure indication, but not a calibrated one yet. However, it is good enough to tune up preamps – a lower noise figure is always better, even if you don't know how low it is.

Noise Source Calibration

Much of the high price of commercial noise sources pays for the NTIS-traceable calibration. Building a noise source only solves part of the problem – now we need to calibrate it.

The basic calibration technique is to measure something with a known noise figure using the new noise source then calculate what ENR would produce the indicated noise figure. Fortunately, the calculation is a simple one involving only addition and subtraction; no fancy computer program required. Simply subtract the indicated noise figure, $NF_{indicated}$, from the known noise figure, NF_{actual}, and add the difference to the ENR for which the meter was calibrated, ENR_{cal} :

$$ENR\ (noise\ source) = ENR_{cal} + (NF_{actual} - NF_{indicated})$$

This procedure must be repeated at each frequency of interest; at least once for each ham band should be fine for amateur use.

The known noise figure is best found by making the measurement with a calibrated noise source, then substituting the new noise source so there is little opportunity for anything to change. Next best would be a sky noise measurement on a preamp. Least accurate would be to measure a preamp at a VHF conference or other remote location, then bring it home and measure it, hoping that nothing rattled loose on the way. If you can't borrow a calibrated noise source, it would be better to take your noise source elsewhere and calibrate it. Perhaps we could measure noise sources as well as preamps at some of these events.

Using the noise source

Now that the ENR of the noise source has been calibrated, the noise figure calibration must be adjusted to match. However, the Model 75 in the *CAL* position has only two dB of adjustment range marked on the meter scale. Older instruments have no adjustment at all. However, we can just turn around the equation we used to calculate the ENR and calculate the NF instead:

$$NF_{actual} = NF_{indicated} + (ENR\ (noise\ source) - ENR_{cal})$$

There is a short cut. My noise source has an ENR around 12 dB, so I set the "CAL ADJ" in the *CAL* position as if the ENR were exactly 3 dB higher, then subtract 3 dB from the reading. Even easier, the meter has a *+3dB* position on the "ADD TO NOISE FIGURE" switch. Using that position, I can read the meter starting at 0 dB. Any ENR difference from 15 dB that matches one of the meter scales would also work } rather than an involved explanation, I'd urge you to do the noise figure calculations, then try the switch positions and see what works best for quick readout.

Reminder: noise figure meters have a very slow time constant, as long as 10 seconds for some of the older models, to smooth out the random nature of noise. *Tune slowly!*

Don't despair if the ENR of your noise source is much less than 15 dB. The optimum ENR is about 1.5 dB higher than the noise figure being measured.[1] The fact that today's solid state noise sources have an ENR around 5 dB rather than the 15 dB of 20 or 30 years ago shows how much receivers have improved.

Conclusion

The value of noise figure measurement capability is to help us all to hear better. A good noise source is an essential part of this capability. Accurate calibration is not necessary, but helps us to know whether our receivers are as good as they could be.

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Reflections on the Smith Chart

Although most radio amateurs have seen the Smith Chart, it is often regarded with trepidation. It is supposed to be complicated and subtle. However, the chart is extremely useful in circuit analysis, especially when transmission lines are involved. The Smith Chart is not limited to transmission-line and antenna problems.

The basis for the chart is Eq 4 in the main text relating reflection coefficient to a terminating impedance. Eq 4 is repeated here:

$$\rho = \frac{Z - Z_0}{Z + Z_0} \quad (1)$$

where Z_0 is the characteristic impedance of the chart, and Z = R + *j* X is a complex terminating impedance. Z might be the feed-point impedance of an antenna connected to a Z_0 transmission line.

It is useful to define a normalized impedance $z = Z/Z_0$. The normalized resistance and reactance become $r = R/Z_0$ and $x = X/Z_0$. Inserting these into Eq 1 yields:

$$\rho = \frac{z - 1}{z + 1} \quad (2)$$

where r and z are both complex, each having a magnitude and a phase when expressed in polar coordinates, or a real and an imaginary part in XY coordinates.

Eq 1 and 2 have some interesting

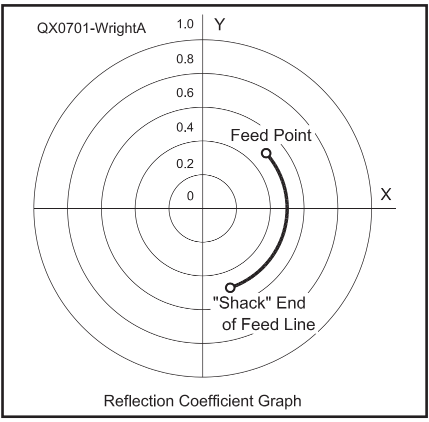

Fig A—Plot of polar reflection coefficient. Circles represent contours of constant ρ. The starting "feed point" value, 0.5 at +45°, represents an antenna impedance of 69.1 + *j* 65.1 Ω with Z_0 = 50 Ω. The arc represents a 15-ft section of 50-Ω, VF 0.66 transmission line at 7 MHz, yielding a shack ρ of 0.5 at –71.3°. The shack z is calculated as 40.3 – *j* 50.9 Ω.

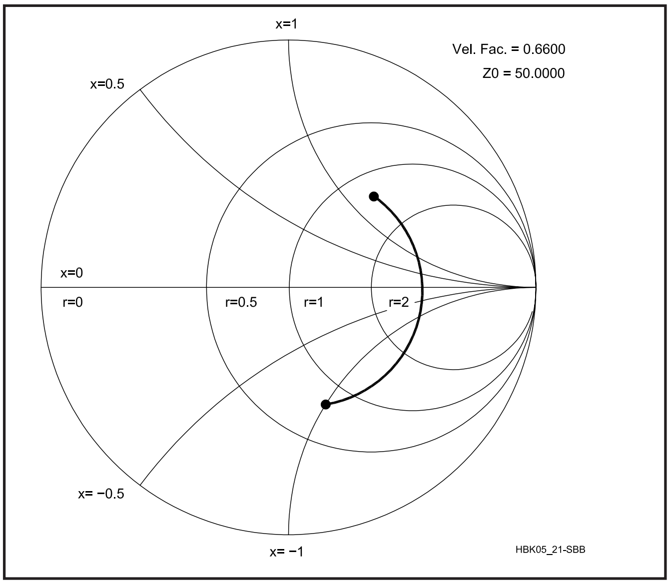

Fig B—This plot shows a Smith Chart. The circles now represent contours of constant normalized resistance or reactance. Note the arc with the markers: This illustrates the same antenna and line used in the previous figure. The plot is the same on the two charts; only the scale details have changed.

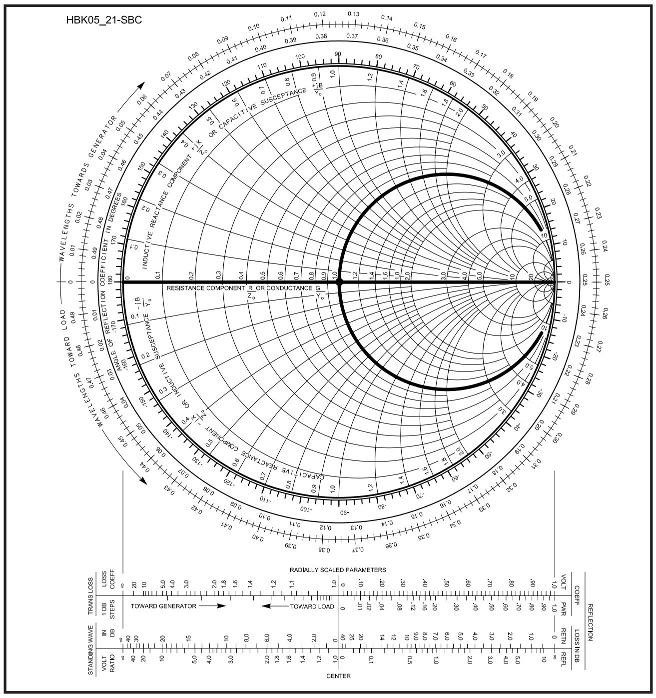

Fig C—The Smith Chart shown in Fig B was computer generated. A much more detailed plot is presented here; this is the chart form used by Smith, suitable for graphic applications. This chart is used with the permission of Analog Instruments.

and useful properties, characteristics that make them physically significant:

- Even though the components of z (and Z) may take on values that are very large, the reflection coefficient ρ, is restricted to always having a magnitude between zero and one if z has a real part, r, that is positive.
- If all possible values for *ρ* are examined and plotted in polar coordinates, they will lie within a circle with a radius of one. This is termed *the unit circle*. A plot is shown in **Fig A**.
- An impedance that is perfectly matched to Z_0, the characteristic value for the chart, will produce a ρ at the center of the unit circle.
- Real Z values, ones that have no reactance, "map" onto a horizontal line that divides the top from the bottom of the unit circle. By convention, a polar variable with an angle of zero is on the x axis, to the right of the origin.
- Impedances with a reactive part produce ρ values away from the dividing line. Inductive impedances with the imaginary part greater than zero appear in the upper half of the chart, while capacitive impedances appear in the lower half.
- Perhaps the most interesting and exciting property of the reflection coefficient is the way it describes the impedance-transforming properties of a transmission line, presented in closed mathematical form in the main text as Eq 11.

Neglecting loss effects, a transmission line of electrical length θ will transform a normalized impedance represented by ρ to another with the same magnitude and a new angle that differs from the original by –2θ. This rotation is clockwise.

Clearly, the reflection coefficient is more than an intermediate step in a mathematical development. It is a useful, alternative description of complex impedance. However, our interest is still focused on impedance; we want to know, for example, what the final z is after transformation with a transmission line. This is the problem that Phillip Smith solved in creating the Smith Chart. Smith observed that the unit circle, a graph of reflection coefficient, could be labeled with lines representing *normalized impedance*. A Smith Chart is shown in **Fig B**. All of the lines on the chart are complete or partial circles representing a line of constant normalized resistance and reactance.

How might we use the Smith Chart? A classic application relates antenna feed-point impedance to the impedance seen at the end of the "shack" end of the line. Assume that the antenna impedance is known, $Z_a = R_a + j X_a$. This complex value is converted to normalized impedance by dividing R_a and X_a by Z_0 to yield $r_a + j x_a$, and is plotted on the chart. A compass is then used to draw an arc of a circle centered at the origin of the chart. The arc starts at the normalized antenna impedance and proceeds in a clockwise direction for 2θ°, where θ is the electrical degrees, derived from the physical length and velocity factor of the transmission line. The end of the arc represents the normalized impedance at the end of the line in the shack; it is denormalized by multiplying the real and imaginary parts by Z_0.

Antenna feed-point Z can also be inferred from an impedance measurement at the shack end of the line. A similar procedure is followed. The only difference is that rotation is now in a counterclockwise direction. The Smith Chart is much more powerful than depicted in this brief summary. A detailed treatment is given by Phillip H. Smith in his classic book: *Electronic Applications of the Smith Chart* (McGraw-Hill, 1969). I also recommend his article "Transmission Line Calculator" in Jan 1939 *Electronics*. Joseph White presented a wonderful summary of the chart in a short but outstanding paper: "The Smith Chart: An Endangered Species?" Nov 1979 *Microwave Journal*. *—Wes Hayward, W7ZOI*

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QST Issue: Mar 1957
Title: Simplified Design of Impedance-Matching Networks, Part I of III
Author: George Grammer, W1DF

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Simplified Design of Impedance-Matching Networks

In Three Parts
Part I—Basic Principles and the L Network

BY GEORGE GRAMMER, W1DF

• Breaking down the design of matching circuits into a step-by-step process results in a method that is not only as simple to understand as anything of this nature ever can be, but in which the desired numerical results are obtained through the use of the most elementary type of arithmetic. The second part of the article will take up the design of pi and T networks and the third will discuss some special applications and practical features.

The general problem of transferring r.f. power from one device to another is one of "matching impedances." This is a term for the process of transforming the resistance of the device that receives the power into a value which the device that furnishes the power wants to have as a load. The power-receiving device may be, for example, a flat 52-ohm line. The power-furnishing device may be the final amplifier tube in a transmitter, and may require a load resistance of say 2000 ohms for delivering the desired amount of power at good efficiency. To transfer the power from the tube to the line, the amplifier output circuit must transform the 52-ohm actual load into a 2000-ohm load as seen by the tube.

The design of such matching circuits or "networks" is surprisingly simple, provided it is broken down into a series of steps. To use the process intelligently, it is necessary to understand the circuit action that gives the resistance transformation, but this is not difficult if the meaning and behavior of reactance are appreciated.[1] Reactance is the key to the operation of practically all r.f. circuits, and without an understanding of it there is little hope of being able to design such circuits.

Resistance and Impedance

The resistances to be matched are seldom actual resistors. The term resistance is used here in its broader interpretation as the voltage-to-current ratio at which power is consumed or transferred. Thus a resonant antenna has a "resistance" of 70 ohms because the current in amperes that flows into its terminals is 1/70 of the number of volts applied to the terminals. A flat 52-ohm line has a resistance of 52 ohms because the current in amperes is equal to 1/52 of the volts applied to the line. Neither the antenna nor the line actually *consumes* power; each simply passes it on to something else. For the purpose of circuit design it is convenient to substitute the resistance symbol for these and similar devices, because their behavior conforms to that of actual resistances.

The term "impedance" is used in a comparable sense. It too is a voltage-to-current ratio. It is a more general term than resistance because it implies that all of the power supplied may not be consumed or passed on, but a certain proportion of it may be returned to the source during some part of the a.c. cycle. When this happens the actual device, be it antenna, transmission line or whatnot, can be represented by a combination of resistance and reactance. The resistive part represents the voltage-to-current ratio at which power is either consumed or passed on; the reactive part the voltage-to-current ratio at which the power is returned to the source.[2]

Determining the values of resistance to be matched is often a more difficult problem than designing the circuit to match them. This question can in no case be ignored, but in the present discussion we shall lay it aside and deal with the subject of matching as such.

Equivalence of Series and Parallel Circuits

The basis for many kinds of impedance matching is the fact that for any circuit consisting of resistance and reactance in series there can be found a circuit consisting of resistance and reactance in parallel that will have exactly the same impedance and phase angle.

Thus the series and parallel circuits of Fig. 1 are exactly equivalent if, when a voltage of fixed magnitude and frequency is applied to either circuit, the same value of current results in both cases, and if the phase between current and voltage is also the same. If the two circuits were concealed in separate boxes, there would be no way to tell which of them actually was connected to the voltage source. This means that a simple series combination of resistance and reactance

[1] The subject of reactance is covered in sufficient detail for this purpose in the section on alternating currents in Chapter 2 of the *Handbook*.

[2] The energy that is stored in the electric or magnetic fields of the reactive elements during part of the a.c. cycle is taken from the fields and restored to the circuit — i.e., the source of power — during a subsequent part of the cycle. This "reactive power" is not consumed anywhere in the system, but simply is handed back and forth between the power source and the reactive elements.

can be lifted out of a more complex circuit and its parallel equivalent substituted for it without in any way affecting the over-all operation of the circuit. It is necessary to specify that the frequency remain fixed, because the reactance values change with a change in frequency.[3]

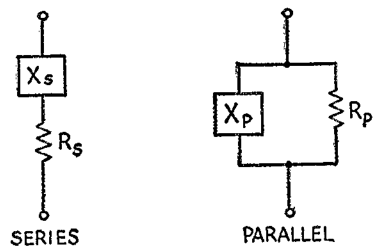


Fig. 1 — Series and parallel circuits containing resistance and reactance. By proper choice of constants, the two circuits will behave identically; i.e., the current and phase angle will be the same in both for the same impressed voltage.

In Fig. 1 the reactances are shown simply as blocks, since the same principles apply whether the reactance is inductive or capacitive. However, if the series reactance, X_S, is inductive the parallel reactance, X_P, in the equivalent parallel circuit also will be inductive, and vice versa. The reactances in such equivalent circuits always are of the same kind. Their values, however, are not identical: that is, X_S is not equal to X_P, and R_S is not equal to R_P. R_S will always be smaller than R_P, and X_S will always be smaller than X_P.

In determining the actual R and X values in the equivalent circuits, it is convenient to introduce the quantity Q. It has the same meaning as the one we ordinarily associate with that letter. That is, in the series circuit

$$Q = \frac{X_S}{R_S} \tag{1A}$$

and in the parallel circuit

$$Q = \frac{R_P}{X_P} \tag{1B}$$

When series and parallel circuits are equivalent, Q has the same value in both.[4]

From ordinary a.c. circuit theory it can be shown that a parallel circuit is equivalent to a given series circuit when

$$R_P = R_S(Q^2 + 1) \tag{2A}$$

$$\text{and } X_P = \frac{R_P}{Q} \tag{2B}$$

while a series circuit is equivalent to a given parallel circuit when

$$R_S = \frac{R_P}{Q^2 + 1} \tag{3A}$$

$$\text{and } X_S = QR_S \tag{3B}$$

When the values of resistance and reactance satisfy these equations the two circuits will have exactly the same impedance and phase angle at the frequency considered.

The significant point in all this is that when the equivalence is achieved, the resistance values are not identical. Herein lies the clue to the matching properties.

Matching by Means of Reactance

Going back to the illustration mentioned earlier, of a 52-ohm load that had to be transformed into 2000 ohms so a tube could deliver its power output to a transmission line, let us assume that 52 ohms (the smaller of the two resistances) corresponds with R_S in Fig. 1. From the preceding discussion we may infer that if a suitable value of reactance, X_S, is added in series with the 52-ohm resistance we can come out with a circuit that is equivalent to a resistance of 2000 ohms in parallel with some value of reactance X_P.

Equation 2A can be rearranged to read

$$\frac{R_P}{R_S} = Q^2 + 1, \tag{4}$$

which says that the ratio of the two resistances, R_P and R_S, corresponds with a specific value of Q, which is

$$Q = \sqrt{\frac{R_P}{R_S} - 1}. \tag{5}$$

This is the relationship we need for matching purposes. In the illustration, R_P/R_S is 2000/52, which is equal to 38.4. Hence to transform 52 ohms into 2000 ohms Q must be equal to

$$Q = \sqrt{38.4 - 1} = \sqrt{37.4} = 6.1$$

The required value of series reactance X_S is found from Equation 3B, and is

$$X_S = 6.1 \times 52 = 318 \text{ ohms}$$

Thus a reactance of 318 ohms in series with the 52-ohm resistive load will make the circuit "look like" a resistance of 2000 ohms (which is what we want) in parallel with a reactance X_P (which we do not want particularly), the value of which is found from Equation 2B:

$$X_P = \frac{2000}{6.1} = 328 \text{ ohms.}$$

The equivalence is shown in Fig. 2. In this figure

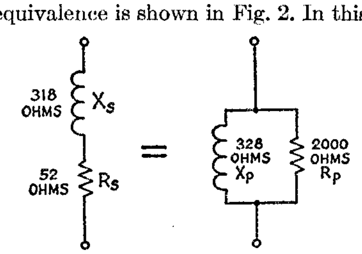


Fig. 2 — An example of series and parallel circuits that are equivalent.

[3] Also, in many practical cases such a substitution might entail a change in accessory circuit details, such as direct-current feed. Obviously, d.c. would not flow through a series capacitor, although it would flow through a resistor in parallel with a capacitor. The discussion here is confined to the alternating-current operation of the circuit.

[4] It is necessary to keep in mind that the Q under consideration is the "operating" Q of the circuit, not the Q of a component, such as a coil. The latter Q is determined by the inherent resistance of the component. In most practical cases the power loss in a component (as represented by its internal resistance) will be very small compared with the power used in the load, so the component resistance can be neglected. The circuit or operating Q is therefore based on the load resistance.

it is assumed that inductive reactance is used, but capacitive reactance of the same numerical value would do equally well.

Since we originally wanted only a *resistive* load of 2000 ohms for the tube, something has to be done about the 328-ohm reactance in parallel with it. Before taking up that question, it may be observed that Fig. 3 gives in graph form the values of Q required for matching any two resistances having a ratio from 1 to 1000. For ratios above 100, the error in dropping the numeral 1 from Equation 5 will be negligible, so the relationship becomes

$$Q = \sqrt{\frac{R_P}{R_S}} \qquad (6)$$

Use of Fig. 3 obviates the necessity for taking the square root called for in Equations 5 or 6. Equations 2B and 3B call for nothing more than simple multiplication and division. It could hardly be said that the process of finding the proper value of reactance is complicated or difficult.

Circuit Action

The physical process by which the resistance transformation takes place poses no mystery. Adding reactance in series with resistance raises the impedance of the circuit, and the total impedance can be increased to any desired value by this method. On the other hand, if we want to develop a given amount of power in R_S, we must put a fixed amount of current through it regardless of the reactance in series. Hence, as the ratio of X_S to R_P (that is, Q) is increased by adding more and more reactance at X_S, more and more voltage is needed to force the same current through the circuit and thereby maintain the same power in R_S.

Suppose, in the illustration, that we want 52 watts in the 52-ohm resistance. In the resistance alone, this would require 52 volts and the current would be 1 ampere. If reactance is now added in series, the voltage must be increased to keep the current at 1 ampere. Eventually, as X_S is made larger, we reach the value of 318 ohms and find that the impedance of the circuit is

$$Z = \sqrt{(318)^2 + (52)^2} = 322 \text{ ohms}$$

To put 1 ampere through this circuit requires 322 volts.

Although the product of 322 volts and 1 ampere is 322 volt-amperes, the actual *power* is still 52 watts, because the reactance does not use up power. Nevertheless, the 52 watts is now being supplied to the *circuit* at 322 volts instead of 52 volts. If a circuit consumes 52 watts at 322 volts, Ohm's Law tells us that the resistance of that circuit should be

$$R = \frac{E^2}{P} = \frac{(322)^2}{52} = 2000 \text{ ohms}$$

On the other hand, a 2000-ohm resistor across a 322-volt source should take only 322/2000 or 0.161 ampere, whereas the actual current through the circuit is 1 ampere. The "excess" current is the current flowing through the parallel reactance, X_P. The current in this reactance has just the right value to make the total current become 1 ampere when combined with the 0.161 ampere flowing in R_P.

The L Section

Demanding that a source of power furnish 322 volt-amperes in order to deliver 52 useful watts would hardly be sporting, so something needs to be done to circumvent this aspect of the otherwise beneficial effect of the series reactance. The solution, which is quite simple, is variously called "power-factor correction," "reactance cancellation," or just "tuning to resonance."

It will be recalled that in a pure capacitance the current is a quarter cycle *ahead* of the applied voltage, while in a pure inductance the current is a quarter cycle *behind* the applied voltage. These currents are numerically equal when the reactances are numerically equal and the same voltage is applied to both. If we place two such reactances across a source of voltage, the leading current through the capacitance just balances the lagging current through the inductance, and if the two reactances are the only circuit elements connected to the voltage source that source

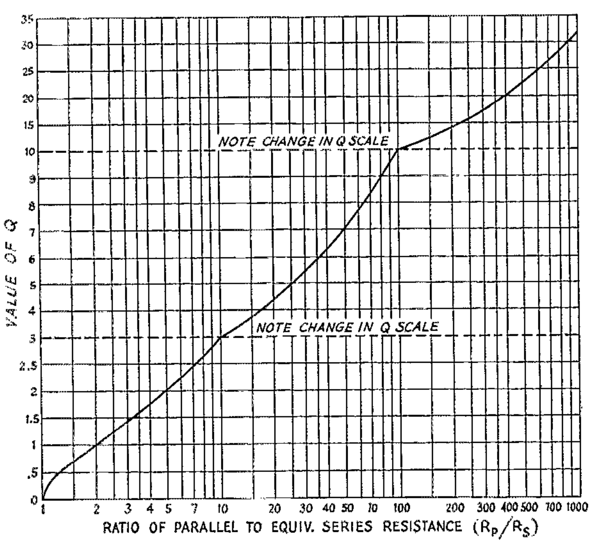


Fig. 3 — Q required for matching resistances having ratios (R_P/R_S) from 1 to 1000.

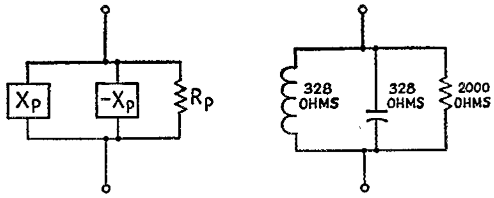


Fig. 4 — Reactance cancellation in the equivalent parallel circuit.

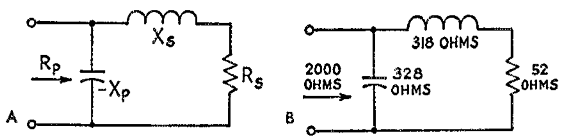
does not even know the reactances exist. In other words, no current flows out of the source even though large currents may be flowing in the capacitance and inductance.[5]

This type of circuit action is just what is needed for getting rid of the unnecessary volt-amperes. By placing a reactance having the same value as X_P, but of the opposite kind, in parallel with X_P all the reactance is effectively eliminated from the equivalent parallel circuit and the resistance alone is left. This is indicated in Fig. 4 by using a minus sign to show that the reactance is of the opposite kind. In the illustration of Fig. 2, where we have 328 ohms of inductive reactance in parallel with 2000 ohms of resistance, it is necessary to add a capacitive reactance of 328 ohms in parallel as shown at the right in Fig. 4. This cancels the inductive reactance and leaves just the 2000-ohm resistance.

Of course the actual circuit we began with is the one at the left in Fig. 2. The parallel equivalent at the right in that figure is just that — an equivalent. It is not the physical circuit even though it exhibits exactly the same impedance and phase angle as the series circuit. So when the compensating reactance, $-X_P$, is added in parallel the resulting physical circuit is as shown in Fig. 5. R_P is now shown with an arrow to indicate that it is the resistance that a power source connected to the terminals would "see." The physical configuration of the illustrative circuit is also shown in Fig. 5. If a flat 52-ohm line were connected to replace the 52-ohm resistor, a power tube connected to the circuit terminals would see the 2000-ohm load it wants, and the power output would be delivered to the transmission line without loss.

This circuit is the "L section," and it develops quite naturally and easily out of the equivalence of simple parallel and series circuits. The process that has just been described is the process of designing an L section to match two resistances. Since the L section is the building block from which more complicated circuits such as the pi and T are constructed, it is necessary to understand it thoroughly before taking the next step.

Fig. 5 — Typical practical circuit corresponding with Fig. 4.



Summary of L Section Design

At this point it is well to summarize the step-by-step process of L-section design because the same procedure is used in any network calculation, whether it is the simple L section or a more complicated type:

1) Given the two resistance values to be matched, place the smaller in the series arm of the circuit (R_S) and the larger in the parallel arm (R_P).

2) Find the ratio R_P/R_S.

3) From Equation 5 — or, when Q is 10 or more, from the simpler form of Equation 6 — find the required Q for matching. Alternatively, use Fig. 3 to find Q.

4) From Equation 3B find X_S. X_S may be either inductive or capacitive. The choice will depend on the purpose for which the circuit is to be used, as discussed below.

5) From Equation 2B find X_P. The reactance used in the actual circuit will be of the opposite type to that chosen for X_S.

These five steps determine all the necessary values, but one more is necessary for arriving at circuit constants:

6) Convert the reactances to inductance and capacitance. The following formulas may be used:

$$L = 0.159\frac{X}{f} \tag{7}$$

$$C = \frac{159{,}000}{fX} \tag{8}$$

where L = inductance in μh.
C = capacitance in $\mu\mu$f.
X = reactance in ohms
f = frequency in megacycles

Choosing the Kind of Reactance

Purely from the standpoint of matching, either inductive or capacitive reactance can be selected for the series arm and the circuit performance will be exactly the same. The circuit of Fig. 5B could be changed to that of Fig. 6, for example, and the tube would still see a purely-resistive load of 2000 ohms. However, in this particular

[5] This is called a "circulating" current, since it is confined to the loop formed by the inductance and capacitance alone. If there is difficulty in visualizing how a current can exist in such a loop with no current coming from the source of energy, it may help to recall that if the inductance and capacitance were perfect (they never are, of course) any energy supplied to them would be passed back and forth between them, in their electric and magnetic fields, without loss and so a current could circulate in such a circuit forever. Hence no continuous supply of current is required from the source. However, the source does have to supply the energy originally. This transfer from the source to the circuit takes place in an initial "transient" state that is not covered in ordinary circuit theory. The latter assumes "steady-state" conditions — i.e., it deals with what goes on after equilibrium has been reached in the circuit.

application no doubt Fig. 5B would be chosen in preference to Fig. 6, for the reason that harmonic suppression would be better with the former circuit. In Fig. 5B harmonics generated by the tube tend to be by-passed through the shunt capacitance, and are choked off from the 52-ohm load by the series inductance. In Fig. 6, they would be more or less forced to flow to the load

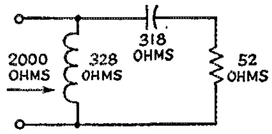


Fig. 6 — **An alternative form giving the same impedance ratio.**

because the inductance does not by-pass them effectively and the capacitance transfers them readily to the load.

In short, the choice frequently is determined by considerations that have nothing to do with impedance matching as such. In each problem, such things as harmonic suppression, d.c. feed, whether one terminal of a condenser may be grounded or whether both terminals must be insulated from ground, and similar points not related to matching impedances should be given consideration in arriving at a decision.

There are times when a free choice is not always possible or convenient, as when one of the resistances to be matched has unavoidable reactance of one kind or the other associated with it. This occurs frequently in antenna problems. Some typical cases will be discussed later.

Efficiency

The reactances in the foregoing discussion have tacitly been assumed to be completely loss-free. While this is never so, the power loss in the circuit itself is small, in the average case, and can be neglected as a factor in the circuit design. Such losses as occur are almost entirely in the coils. Air condensers, at least at frequencies below 30 Mc., have extremely low losses.

The power loss in a coil depends upon the inherent Q of the coil — that is, the ratio of coil reactance to coil resistance. (This is not the Q figure used in the calculations described above; the latter is the "circuit" or "operating" Q. See Footnote 4.) In circuits handling appreciable power, the coils are generally of good-enough construction to have Q's of the order of 200 or more. If the coil in the circuit of Fig. 5B has a Q of 200, its effective resistance is X/Q, or 318/200. This is approximately 1.6 ohm. For higher accuracy in designing the circuit the coil resistance should be added to the load resistance to find the actual resistance in the load circuit. In most cases this is an unnecessary refinement because the coil resistance usually will be but a small percentage of the total resistance, and the tuning elements usually can be varied over enough of a range to compensate for even greater discrepancies than are likely to arise from this cause.

The efficiency of the circuit is the ratio of the power consumed in the load to the power put into the circuit by the source. It will be equal to the ratio of the actual load resistance to the total resistance, considering the series arm of the circuit. In the example the efficiency is

$$\frac{52}{52 + 1.6} = 0.97 \text{ or } 97 \text{ per cent}$$

when the coil resistance is included. Other cases might not be so favorable; in general, the efficiency will decrease if the coil Q is decreased and if the circuit Q is increased (increasing R_P/R_S). However, an L section uses the minimum possible circuit Q for matching, and so is inherently the most efficient type of matching circuit.

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QST Issue: Apr 1957
Title: Simplified Design of Impedance-Matching Networks, Part II of III
Author: George Grammer, W1DF

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Simplified Design of Impedance-Matching Networks

In Three Parts — Part II, Pi and T Networks*

BY GEORGE GRAMMER, W1DF

• In this second part it is shown how pi and T networks can be formed quite simply from the basic L network discussed in Part I.

THE L SECTION has the advantages of great simplicity and optimum efficiency, but it is an inflexible arrangement in that only one set of constants will give the required match. The circuit Q, for example, cannot be chosen at will because it is fixed by the ratio of the two resistances. When an amplifier tube is being matched to a load, it is often just as important to have the proper operating Q in the tank as it is to have the proper loading.[6] Also, there are cases where the presence of reactance in the load makes it desirable to use a more complex network.

Two L sections can be combined to form either a pi or T network. Both the pi and T are useful arrangements for increasing the circuit Q and thus improving selectivity, and for increasing flexibility in adjustment.

The design methods used for these — or any other — combinations are practically identical. Basically there is just one method — the one used for designing the L section.

The Pi Network

Of all the possible configurations, the pi network is probably the one most familiar to amateurs because of its application as the tank circuit in r.f. power amplifiers. It lends itself to simple design methods when it is considered as two "back-to-back" L networks constructed to match an assumed "virtual" resistance at the meeting point.[7]

The principle is shown in Fig. 7. R_{P1} is the load (power-receiving device) to be matched to the

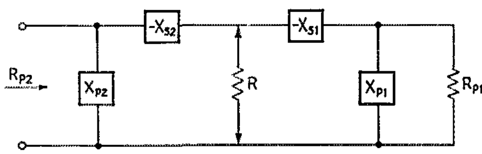

Fig. 7 — Development of the pi network from two back-to-back L networks.

source of power, which requires a load R_{P2}. R_{P1} is the parallel resistance of the L section consisting of X_{P1} and $-X_{S1}$, while R is the virtual resistance and corresponds to R_S in the circuits discussed in Part I. Using R as the load, a second L section is formed by $-X_{S2}$ and X_{P2} to transform R to the desired value of resistance for the power source to see. Each section is calculated separately by exactly the method described previously. The virtual resistance R must be smaller than either R_{P1} or R_{P2}, since it is in the series arm of each section, but aside from this may be chosen at will or determined from other circuit requirements. The principal one of such requirements, in the case of an r.f. amplifier tank circuit, will be the desired operating Q.

The design of such a circuit may be shown by continuing with the same resistance values used in the earlier example: a 52-ohm resistive load and a tube requiring a load of 2000 ohms. Let us now impose the requirement that the tank Q (Q_2) be 12. From Equation 2B,

$$X_{P2} = \frac{2000}{12} = 167 \text{ ohms},$$

and from Equation 3A,

$$R = \frac{2000}{(12)^2 + 1} = 13.8 \text{ ohms}.$$

This is smaller than either of the two resistances being matched, and so is a proper value. Using Equation 3B,

$$X_{S2} = 12 \times 13.8 = 166 \text{ ohms}.$$

This completes the design of the L section on the input side of the network. Note that since Q_2 is greater than 10, X_{P2} and X_{S2} are practically equal (although they must be of opposite types), which simply means that this much of the network is identical with an ordinary parallel-resonant tank circuit of the same operating Q.

We can now find the required Q (Q_1) in the output section, since R and R_{P1} are known. Their ratio is $52/13.8 = 3.77$, so from Equation 5 the required Q is

$$Q_1 = \sqrt{3.77 - 1} = 1.66.$$

The same result could be found from Fig. 3. From Equation 3B,

$$X_{S1} = 1.66 \times 13.8 = 23 \text{ ohms}$$

and from Equation 2B,

$$X_{P1} = \frac{52}{1.66} = 31.3 \text{ ohms}.$$

The network design is now complete and is shown in Fig. 8A. The virtual resistance does not

* Part I of this article appeared in March, 1957, *QST*.
[6] For discussion of tank-circuit Q, see chapter on transmitters in *The Radio Amateur's Handbook*.
[7] Bruene, "Pi-Network Calculator", *Electronics*, May, 1945.

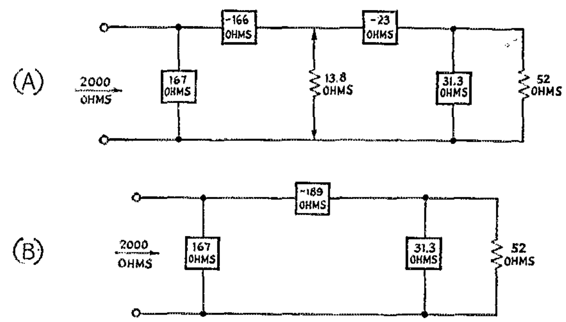

Fig. 8 — Combining L-section elements into the pi configuration.

actually appear in the circuit, of course, so X_{S1} and X_{S2} can be added together and only one physical component is required. In this example it has a reactance of 166 + 23 = 189 ohms. The minus sign in Fig. 8A is used only to indicate that the reactance is of the opposite type to that chosen for X_{P1} and X_{P2}.[8]

Reactance Combinations

The design so far is quite general and we are free to choose types of reactance as we please, within the restriction that opposite types must be used in the two arms of each L section. There are four possibilities, as shown in Fig. 9, each leading to a different-looking final circuit but all providing the proper match between the two resistances. Fig. 9A is the best one for matching a tube to its load because it utilizes the components to best advantage in suppressing harmonics. Fig. 9B is a high-pass configuration and would be undesirable in a tank circuit. Note that in C and D the reactance of the connecting element in the practical circuit at the right is the *difference* between the reactances of the two series elements of the individual L sections, since opposite kinds of reactance are used in the series elements in these two cases.

The four circuits are shown primarily to illustrate the variety of networks, all of different appearance in the final version, that can be formed from simple L sections having fundamentally the same constants. They are alike only in that each will match 52 ohms to 2000 ohms. In other respects, such as relative suppression of frequencies higher and lower than the operating frequency, they are not identical. In some applications one or another of them might be preferable to Fig. 9A, but a decision cannot be made on this point until the nature of the problem is known.

[8] In particular, the use of the negative sign in this and similar examples should not be confused with the use of the same sign to indicate capacitive reactance. The method of calculation described here does not require such use of signs nor is it necessary to bring the operator j into the calculations.

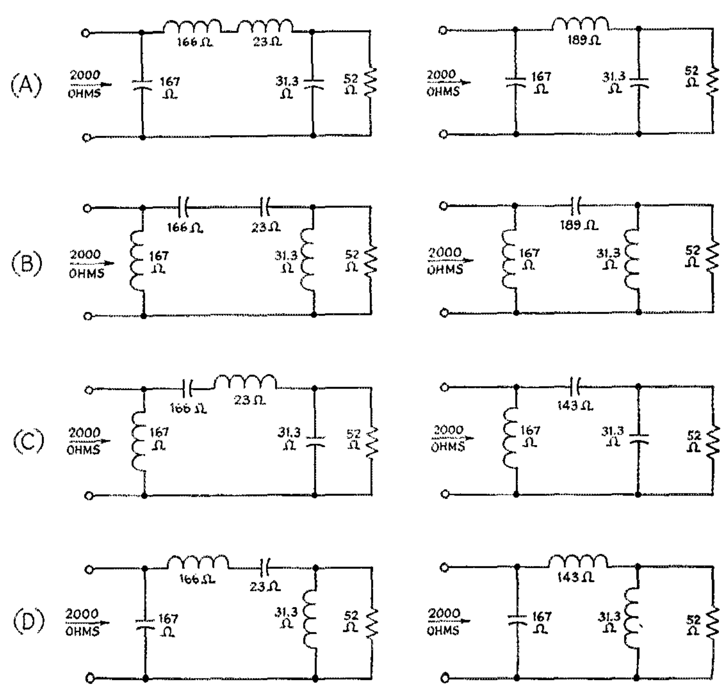

Fig. 9 — The four possible combinations of L-sections into pi networks. Drawings at the left show the basic L sections; those at the right show the final forms.

Summary of Pi Network Design

1) Break the proposed network into two L sections as shown in Fig. 7.

2a) If the input section is to have a specified value of Q, use Equation 3A to find R. The value of R must be smaller than either of the two resistances to be matched. If it is not, a higher value of Q must be used.

2b) Or, if there are no restrictions on the Q of the input section, select some value for R that is less than either of the two resistances to be matched.

3) Calculate the constants of the input L section to match the desired resistance, R_{P2}, to R.

4) Calculate the constants of the output section to match R to the other desired resistance, R_{P1}.

5) Add the values of the reactances in the series arms of the L sections (X_{S1} and X_{S2}) to find the value of the series reactance in the pi network.

6) Convert the final reactance values to inductance and capacitance.

Covering a Band

The reactance values obtained from the design method apply at any frequency, so it is a simple matter to determine the range of variation that must be supplied to cover an amateur band. For example, using Equations 7 and 8 with the right-hand circuit of Fig. 9A for 3500 and 4000 kc. will give the following values for L and C:

Reactance	*3500 kc.*	*4000 kc.*
31.3 ohms (C)	1450 $\mu\mu$f.	1270 $\mu\mu$f.
167 ohms (C)	272 $\mu\mu$f.	238 $\mu\mu$f.
189 ohms (L)	8.57 μh.	7.5 μh.

Note that all three tuning elements must be continuously variable within these limits to produce a match at any frequency in the band while maintaining a constant Q of 12.

Only two elements need be continuously variable to give a match if the consequent variation in Q is permissible. In this event the network should be designed for a minimum value of Q. It is convenient to use a fixed value of inductance and adjust the capacitances to achieve a match, in which case the minimum Q will occur at the high-frequency end of the band. The inductance should be chosen accordingly. In the example above this would mean that an inductance value of 7.5 μh. should be chosen. The problem then is to determine the capacitances required for matching at 3500 kc., in order to establish the necessary range. Although the simplified design method under discussion will not give an exact solution in this case,[9] because there is no specific way of finding the individual values of X_{S1} and X_{S2} when their sum is fixed, a close-enough approximation will result if we assume that the Q of the network on the input side will be inversely proportional to frequency. Thus the Q at 3500 kc. will be 4000/3500 times the Q (12) at 4000 kc., or a Q of 13.7. Then

$X_{P2} = 2000/13.7 = 146$ ohms; $= X_{S2}$, approximately.

$R = 146/13.7 = 10.6$ ohms

$R_{P1}/R = 52/10.6 = 4.88$

$Q_1 = \sqrt{4.88 - 1} = 1.97$

$X_{P1} = 52/1.97 = 26.4$ ohms

$X_{S1} = 10.6 \times 1.97 = 20.9$ ohms.

The accuracy can be checked by adding X_{S1} and X_{S2}, the sum being 167 ohms, and comparing this with the reactance of the 7.5-μh. coil at 3500 kc. This is 165 ohms, which is amply close agreement. The capacitance values corresponding with 146 ohms and 26.4 ohms at 3500 kc. are, respectively, 311 $\mu\mu$f. and 1720 $\mu\mu$f. These values differ considerably from those of the constant-Q network tabulated earlier.

T Networks

Fundamentally the same method is used for constructing T networks, the difference being that the two back-to-back L networks have their *parallel*-reactance sides joined together. This requires that the virtual resistance be *higher* than either of the two resistances to be matched. The T is often a convenient form when low values of resistance are to be matched, and when for some reason — e.g., suppression of off-frequency radiations such as harmonics — a higher Q is needed than would be provided by a simple L network.

Fig. 10 is the basic circuit used for the T; this circuit may be compared with Fig. 7 for the pi.

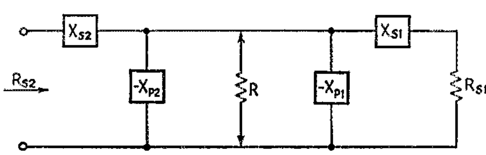

Fig. 10 — T network formed from two L sections.

Each L network is calculated in the same way as in the previous examples. As an illustration, suppose that the load, R_{S1}, is 10 ohms and is to be transformed into 50 ohms (R_{S2}) at the network terminals. Then R may have any desired value larger than 50 ohms. If we choose to make the Q (Q_1) of the first L section 10, then X_{S1} is

---

[9] Somewhat more complicated formulas must be used for finding the exact values when the series inductance (not reactance) is constant. They are

$$X_{P1} = \frac{-R_{P1}X}{R_{P1} \pm \sqrt{R_{P1}R_{P2} - X^2}}$$

$$X_{P2} = \frac{-R_{P2}X}{R_{P2} \pm \sqrt{R_{P1}R_{P2} - X^2}}$$

where X is the total reactance of the series inductance, Fig. 9A. For Fig. 9A, use the plus signs in the denominators, in which event the minus signs in the numerators indicate that the shunt reactances are capacitive. In the general case, the same sign must be chosen for both denominators, but X may be either positive or negative. This leads to the four possible combinations shown in Fig. 9. For further details see W. L. Everitt, *Communication Engineering*, McGraw-Hill Book Co., New York.

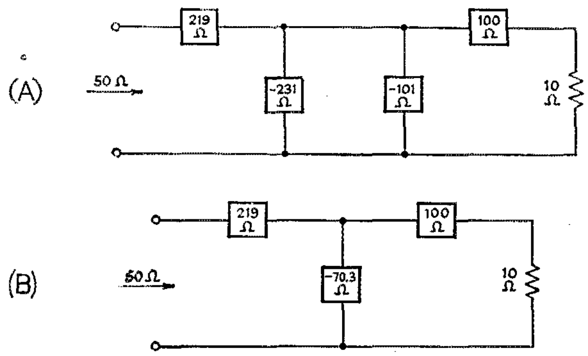

Fig. 11 — Combining L-section elements into the T configuration. Note that since the shunt element is formed from two elements in parallel the resultant reactance is less than either alone

$10R_{S1} = 10 \times 10 = 100$ ohms. From Equation 2A,

$R = 10\,(10^2 + 1) = 10 \times 101 = 1010$ ohms

and from Equation 2B,

$$X_{P1} = \frac{1010}{10} = 101 \text{ ohms.}$$

The Q required for matching R to R_{S2} is found from Equation 5:

$$Q_2 = \sqrt{\frac{1010}{50} - 1} = \sqrt{19.2} = 4.38$$

so

$$X_{P2} = \frac{R}{Q_2} = \frac{1010}{4.38} = 231 \text{ ohms}$$

and

$$X_{S2} = Q_2 R_{S2} = 4.38 \times 50 = 219 \text{ ohms}$$

The elements of the complete network are shown in Fig. 11, which compares with Fig. 8. The choice of signs is again arbitrary, since the only requirement is that the signs be opposite for the two elements of each L network. In Fig. 11A the parallel or shunt elements have been chosen with the same sign, and so can be combined into a single element, as shown in Fig. 11B.

If different signs are chosen for the reactances, there again result four possible combinations as shown in Fig. 12. As compared with the pi configurations in Fig. 9, the reactances that can be combined are in parallel instead of series, and so the net reactance is not given by simple algebraic addition. However, it is easily found: it is equal to the product of the two reactances divided by their sum, if they are of the same kind (i.e., both capacitive or both inductive); or to the product divided by their *difference*, if they are of opposite kinds. In the latter case, the net reactance has the same sign as the smaller of the two; in Fig. 12C, for example, the capacitive reactance, 101 ohms, is smaller than the inductive reactance, 231 ohms, and so the net reactance, 179 ohms, is capacitive. The opposite is true in Fig. 12D.

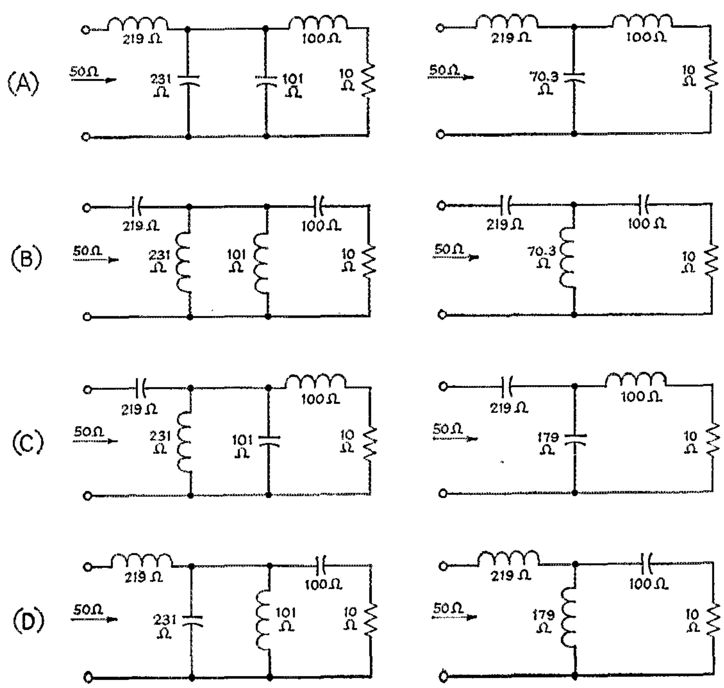

Fig. 12 — The four possible combinations of L sections into T networks. The development is shown at the left; final forms at the right.

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Author: George Grammer, W1DF

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Simplified Design of Impedance-Matching Networks

In Three Parts — Part II, Pi and T Networks*

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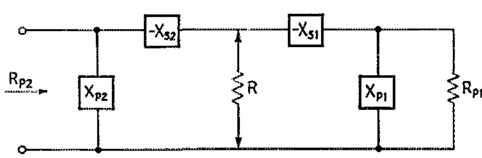

Fig. 7 — Development of the pi network from two back-to-back L networks.

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The design of such a circuit may be shown by continuing with the same resistance values used in the earlier example: a 52-ohm resistive load and a tube requiring a load of 2000 ohms. Let us now impose the requirement that the tank Q (Q_2) be 12. From Equation 2B,

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$$X_{S2} = 12 \times 13.8 = 166 \text{ ohms}.$$

This completes the design of the L section on the input side of the network. Note that since Q_2 is greater than 10, X_{P2} and X_{S2} are practically equal (although they must be of opposite types), which simply means that this much of the network is identical with an ordinary parallel-resonant tank circuit of the same operating Q.

We can now find the required Q (Q_1) in the output section, since R and R_{P1} are known. Their ratio is 52/13.8 = 3.77, so from Equation 5 the required Q is

$$Q_1 = \sqrt{3.77 - 1} = 1.66.$$

The same result could be found from Fig. 3. From Equation 3B,

$$X_{S1} = 1.66 \times 13.8 = 23 \text{ ohms}$$

and from Equation 2B,

$$X_{P1} = \frac{52}{1.66} = 31.3 \text{ ohms}.$$

The network design is now complete and is shown in Fig. 8A. The virtual resistance does not

* Part I of this article appeared in March, 1957, *QST*.
[6] For discussion of tank-circuit Q, see chapter on transmitters in *The Radio Amateur's Handbook*.
[7] Bruene, "Pi-Network Calculator", *Electronics*, May, 1945.

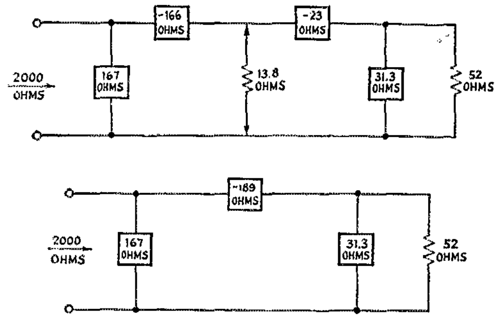

Fig. 8 — Combining L-section elements into the pi configuration.

actually appear in the circuit, of course, so X_{S1} and X_{S2} can be added together and only one physical component is required. In this example it has a reactance of 166 + 23 = 189 ohms. The minus sign in Fig. 8A is used only to indicate that the reactance is of the opposite type to that chosen for X_{P1} and X_{P2}.[8]

Reactance Combinations

The design so far is quite general and we are free to choose types of reactance as we please, within the restriction that opposite types must be used in the two arms of each L section. There are four possibilities, as shown in Fig. 9, each leading to a different-looking final circuit but all providing the proper match between the two resistances. Fig. 9A is the best one for matching a tube to its load because it utilizes the components to best advantage in suppressing harmonics. Fig. 9B is a high-pass configuration and would be undesirable in a tank circuit. Note that in C and D the reactance of the connecting element in the practical circuit at the right is the *difference* between the reactances of the two series elements of the individual L sections, since opposite kinds of reactance are used in the series elements in these two cases.

The four circuits are shown primarily to illustrate the variety of networks, all of different appearance in the final version, that can be formed from simple L sections having fundamentally the same constants. They are alike only in that each will match 52 ohms to 2000 ohms. In other respects, such as relative suppression of frequencies higher and lower than the operating frequency, they are not identical. In some applications one or another of them might be preferable to Fig. 9A, but a decision cannot be made on this point until the nature of the problem is known.

[8] In particular, the use of the negative sign in this and similar examples should not be confused with the use of the same sign to indicate capacitive reactance. The method of calculation described here does not require such use of signs nor is it necessary to bring the operator j into the calculations.

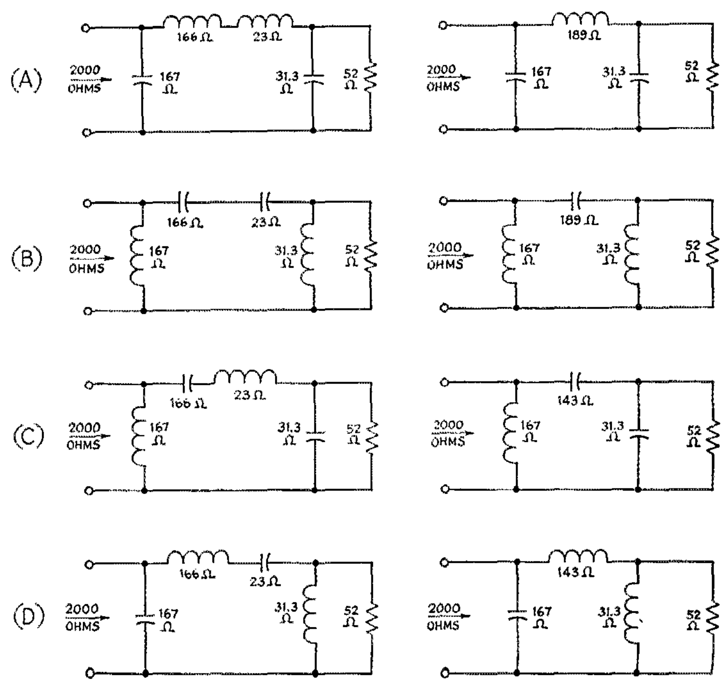

Fig. 9 — The four possible combinations of L-sections into pi networks. Drawings at the left show the basic L sections; those at the right show the final forms.

Summary of Pi Network Design

1) Break the proposed network into two L sections as shown in Fig. 7.

2a) If the input section is to have a specified value of Q, use Equation 3A to find R. The value of R must be smaller than either of the two resistances to be matched. If it is not, a higher value of Q must be used.

2b) Or, if there are no restrictions on the Q of the input section, select some value for R that is less than either of the two resistances to be matched.

3) Calculate the constants of the input L section to match the desired resistance, R_{P2}, to R.

4) Calculate the constants of the output section to match R to the other desired resistance, R_{P1}.

5) Add the values of the reactances in the series arms of the L sections (X_{S1} and X_{S2}) to find the value of the series reactance in the pi network.

6) Convert the final reactance values to inductance and capacitance.

Covering a Band

The reactance values obtained from the design method apply at any frequency, so it is a simple matter to determine the range of variation that must be supplied to cover an amateur band. For example, using Equations 7 and 8 with the right-hand circuit of Fig. 9A for 3500 and 4000 kc. will give the following values for L and C:

Reactance	*3500 kc.*	*4000 kc.*
31.3 ohms (C)	1450 $\mu\mu$f.	1270 $\mu\mu$f.
167 ohms (C)	272 $\mu\mu$f.	238 $\mu\mu$f.
189 ohms (L)	8.57 μh.	7.5 μh.

Note that all three tuning elements must be continuously variable within these limits to produce a match at any frequency in the band while maintaining a constant Q of 12.

Only two elements need be continuously variable to give a match if the consequent variation in Q is permissible. In this event the network should be designed for a minimum value of Q. It is convenient to use a fixed value of inductance and adjust the capacitances to achieve a match, in which case the minimum Q will occur at the high-frequency end of the band. The inductance should be chosen accordingly. In the example above this would mean that an inductance value of 7.5 μh. should be chosen. The problem then is to determine the capacitances required for matching at 3500 kc., in order to establish the necessary range. Although the simplified design method under discussion will not give an exact solution in this case,[9] because there is no specific way of finding the individual values of X_{S1} and X_{S2} when their sum is fixed, a close-enough approximation will result if we assume that the Q of the network on the input side will be inversely proportional to frequency. Thus the Q at 3500 kc. will be 4000/3500 times the Q (12) at 4000 kc., or a Q of 13.7. Then

$X_{P2} = 2000/13.7 = 146$ ohms; $= X_{S2}$, approximately.

$R = 146/13.7 = 10.6$ ohms

$R_{P1}/R = 52/10.6 = 4.88$

$Q_1 = \sqrt{4.88 - 1} = 1.97$

$X_{P1} = 52/1.97 = 26.4$ ohms

$X_{S1} = 10.6 \times 1.97 = 20.9$ ohms.

The accuracy can be checked by adding X_{S1} and X_{S2}, the sum being 167 ohms, and comparing this with the reactance of the 7.5-μh. coil at 3500 kc. This is 165 ohms, which is amply close agreement. The capacitance values corresponding with 146 ohms and 26.4 ohms at 3500 kc. are, respectively, 311 $\mu\mu$f. and 1720 $\mu\mu$f. These values differ considerably from those of the constant-Q network tabulated earlier.

T Networks

Fundamentally the same method is used for constructing T networks, the difference being that the two back-to-back L networks have their *parallel*-reactance sides joined together. This requires that the virtual resistance be *higher* than either of the two resistances to be matched. The T is often a convenient form when low values of resistance are to be matched, and when for some reason — e.g., suppression of off-frequency radiations such as harmonics — a higher Q is needed than would be provided by a simple L network.

Fig. 10 is the basic circuit used for the T; this circuit may be compared with Fig. 7 for the pi.

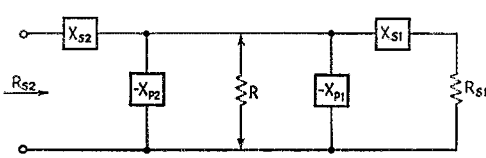

Fig. 10 — T network formed from two L sections.

Each L network is calculated in the same way as in the previous examples. As an illustration, suppose that the load, R_{S1}, is 10 ohms and is to be transformed into 50 ohms (R_{S2}) at the network terminals. Then R may have any desired value larger than 50 ohms. If we choose to make the Q (Q_1) of the first L section 10, then X_{S1} is

[9] Somewhat more complicated formulas must be used for finding the exact values when the series inductance (not reactance) is constant. They are

$$X_{P1} = \frac{-R_{P1}X}{R_{P1} \pm \sqrt{R_{P1}R_{P2} - X^2}}$$

$$X_{P2} = \frac{-R_{P2}X}{R_{P2} \pm \sqrt{R_{P1}R_{P2} - X^2}}$$

where X is the total reactance of the series inductance, Fig. 9A. For Fig. 9A, use the plus signs in the denominators, in which event the minus signs in the numerators indicate that the shunt reactances are capacitive. In the general case, the same sign must be chosen for both denominators, but X may be either positive or negative. This leads to the four possible combinations shown in Fig. 9. For further details see W. L. Everitt, *Communication Engineering*, McGraw-Hill Book Co., New York.

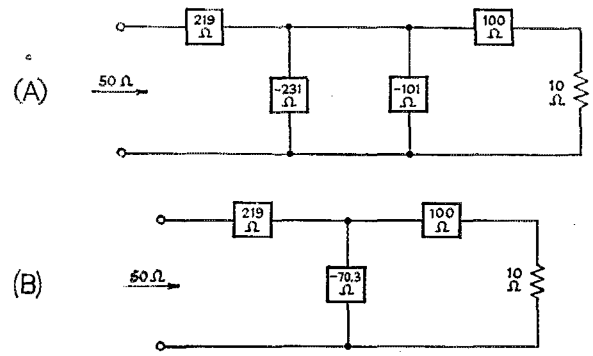

Fig. 11 — Combining L-section elements into the T configuration. Note that since the shunt element is formed from two elements in parallel the resultant reactance is less than either alone

$10R_{S1} = 10 \times 10 = 100$ ohms. From Equation 2A,

$R = 10\,(10^2 + 1) = 10 \times 101 = 1010$ ohms

and from Equation 2B,

$$X_{P1} = \frac{1010}{10} = 101 \text{ ohms.}$$

The Q required for matching R to R_{S2} is found from Equation 5:

$$Q_2 = \sqrt{\frac{1010}{50} - 1} = \sqrt{19.2} = 4.38$$

so

$$X_{P2} = \frac{R}{Q_2} = \frac{1010}{4.38} = 231 \text{ ohms}$$

and

$$X_{S2} = Q_2 R_{S2} = 4.38 \times 50 = 219 \text{ ohms}$$

The elements of the complete network are shown in Fig. 11, which compares with Fig. 8. The choice of signs is again arbitrary, since the only requirement is that the signs be opposite for the two elements of each L network. In Fig. 11A the parallel or shunt elements have been chosen with the same sign, and so can be combined into a single element, as shown in Fig. 11B.

If different signs are chosen for the reactances, there again result four possible combinations as shown in Fig. 12. As compared with the pi configurations in Fig. 9, the reactances that can be combined are in parallel instead of series, and so the net reactance is not given by simple algebraic addition. However, it is easily found: it is equal to the product of the two reactances divided by their sum, if they are of the same kind (i.e., both capacitive or both inductive); or to the product divided by their *difference*, if they are of opposite kinds. In the latter case, the net reactance has the same sign as the smaller of the two; in Fig. 12C, for example, the capacitive reactance, 101 ohms, is smaller than the inductive reactance, 231 ohms, and so the net reactance, 179 ohms, is capacitive. The opposite is true in Fig. 12D.

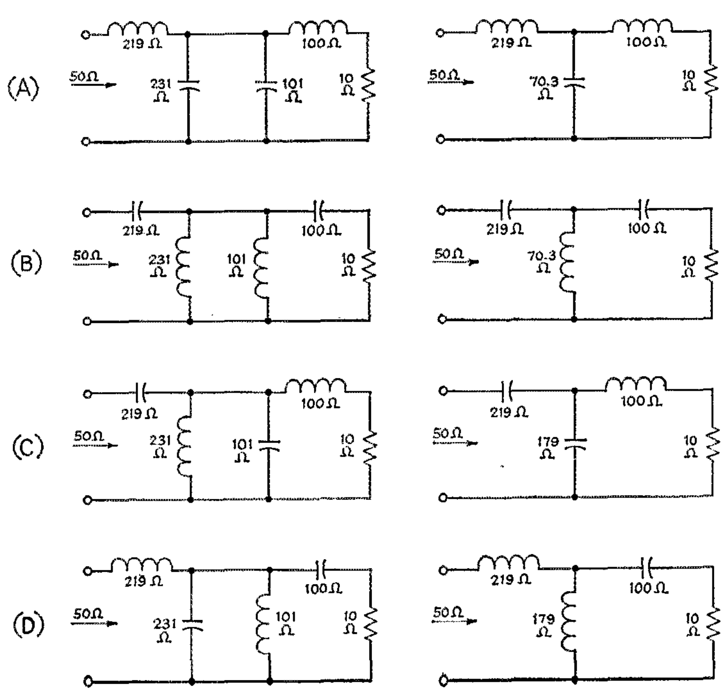

Fig. 12 — The four possible combinations of L sections into T networks. The development is shown at the left; final forms at the right.

Tuned (Resonant) Networks

(excerpted from Chapter 14 of the ARRL Handbook, 2009 and previous editions)

There is a large class of LC networks that utilize resonance at a single frequency to transform impedances over a narrow band. In many applications the circuitry that the network connects to has internal reactances, inductive or capacitive, combined with resistance. We want to absorb these reactances, if possible, to become an integral part of the network design. By looking at the various available network possibilities we can identify those that will do this at one or both ends of the network. Some networks must operate between two different values of resistance, others can also operate between equal resistances. As mentioned before, nearly all networks also allow a choice of selectivity, or Q, where Q is (approximately) the resonant frequency divided by the 3-dB bandwidth.

As a simple example that illustrates the method, consider the generator and load of **Fig 14.58A**. We want to absorb the 20 pF and the 0.1 µH into the network. We use the formulas to calculate L and C for a 500 Ω to 50 Ω L-network, then subtract 20 pF from C and 0.1 µH from L. As a second iteration we can improve the design by considering the resistance of the L that we just found. Suppose it is 2 Ω. We can recalculate new values L′ and C′ for a network from 500 Ω to 52 Ω, as shown in Fig 14.58B.

Further iterations are possible but usually trivial. More complicated networks and more difficult problems can use a computer to expedite absorbing process. Always try to absorb an inductance into a network L and a capacitance into a network C in order to minimize spurious LC resonances and undesired frequency responses. Inductors and capacitors can be combined in series or in parallel as shown in the example. Fig 14.58C shows useful formulas to convert series to parallel and vice versa to help with the designs.

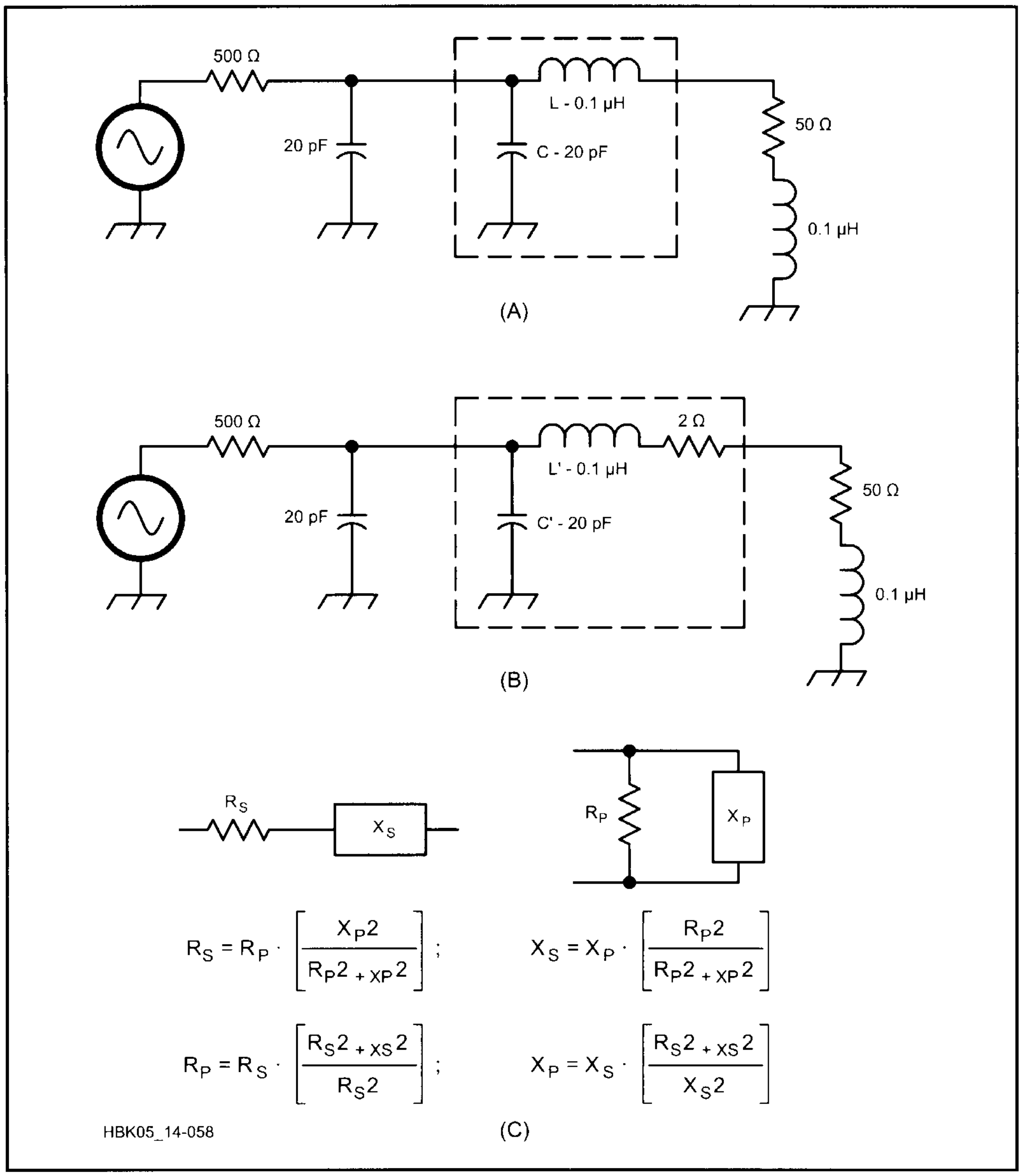

Fig 14.58 — At A, impedance transformation, first iteration. At B, second iteration compensates L and C values for coil resistance. At C, series-parallel conversions.

A set of 14 simple resonant networks, and their equations, is presented in **Fig 14.59**. Note that in these diagrams RS is the low impedance side and RL is the high impedance side and that the X values are calculated in the top-down order given. The program *MATCH.EXE* can perform the calculations.

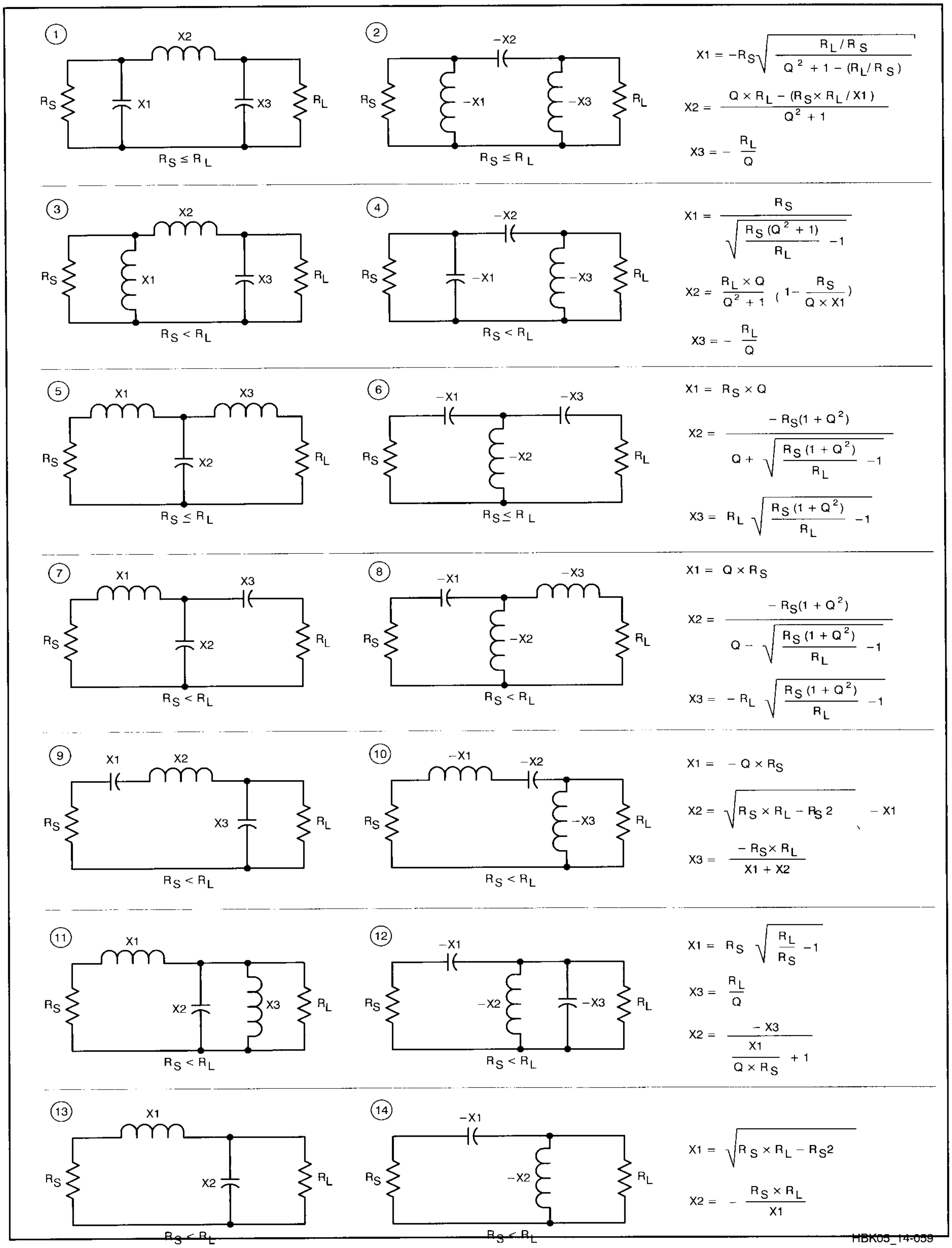

Fig 14.59 — Fourteen impedance transforming networks with their design equations (for lossless components).

Circuit simulation programs can also help a lot with special circuit-design problems and some approaches to resonant network design. It can graph the frequency response, compute insertion loss and also tune the capacitances and inductances across a frequency band. You may select the selectivity (Q) in such programs based on frequency-response requirements. The program can also be trimmed to help realize realistic or standard component values. A math program such as *Mathcad* can also make this a quick and easy process.