A Tutorial on the Decibel

This tutorial combines information from several authors, including Bob DeVarney, W1ICW; Walter Bahnzaf, WB1ANE; and Ward Silver, NØAX

Decibels are part of many questions in the question pools for all three Amateur Radio license classes and are widely used throughout radio and electronics. This tutorial provides background information on the decibel, how to perform calculations involving decibels, and examples of how the decibel is used in Amateur Radio.

The Quick Explanation

- The *decibel (dB)* is a ratio of two power values – see the table showing how decibels are calculated. It is computed using logarithms so that very large and small ratios result in numbers that are easy to work with.
- A positive decibel value indicates a ratio greater than one and a negative decibel value indicates a ratio of less than one. Zero decibels indicates a ratio of exactly one. See the table for a list of easily remembered decibel values for common ratios.
- A letter following dB, such as dBm, indicates a specific reference value. See the table of commonly used reference values.
- If given in dB, the gains (or losses) of a series of stages in a radio or communications system can be added together:

$$System\,Gain\,(\mathrm{dB}) = Gain_1 + Gain_2 + \ldots + Gain_n$$

Losses are included as negative values of gain. i.e. A loss of 3 dB is written as a gain of -3 dB.

Decibels – the History

The need for a consistent way to compare signal strengths and how they change under various conditions is as old as telecommunications itself. The original unit of measurement was the "Mile of Standard Cable." It was devised by the telephone companies and represented the signal loss that would occur in a mile of standard telephone cable (roughly #19 AWG copper wire) at a frequency of around 800 Hz. If you were measuring loss in a telephone line in the early 20th century, you might say that it amounted to "5 Miles of Standard Cable." Since everyone knew how much signal was lost in 1 mile of cable, the effect of a 5-mile loss was easy to understand.

In the 1920s, this unit of measure was replaced by the *Bel (B)* in honor of Alexander Graham Bell, inventor of the telephone and founder of the Bell Telephone Company. One Bel represented a 10-fold gain or loss of power. This turned out to be too much change for most measurements and calculations, so the *decibel (dB)*, or 1⁄10 of a Bel, became the widely used measure of signal change. (The metric prefix *deci-* or *d-* represents 1/10th or multiplication by 0.1.)

Uses of Decibels

Sound intensity or *sound pressure level (SPL)* is also specified in dB. In this case, the reference level of 0 dB corresponds to a pressure of 0.0002 microbars which is the standard threshold for being able to hear a sound. As the sounds get louder, the value of SPL in dB also increases, indicating an increase with respect to the reference level. SPL in the average home is about 50 dB above the 0 dB threshold that serves as the SPL reference. When a vacuum cleaner one meter away is on, SPL increases to 70 dB. A chainsaw one meter away produces a SPL of 110 dB and the threshold of discomfort from sound intensity is 120 dB. Since each 10 dB (or 1 Bel) represents a factor of ten difference, 120 dB (12 Bels) represents a pressure 10^{12} times greater than the reference threshold level – a change of a million-million! Our ears respond logarithmically to changes in sound level, which makes the "decibel" a very useful tool of comparison.

Radio and electronic circuits also deal with signal levels that change by many orders of magnitude. Thus, the decibel is a common feature of the technical side of Amateur Radio. For example, received signal strengths on the HF bands are usually reported in S units. Each S unit represents a change in strength of 5 to 6 dB. Although most receiver S meters are not accurately calibrated, it is useful to consider that a change in signal strength of one S unit is a change in signal power of approximately four.

Here are some other places you'll find the ubiquitous "dee-bee":

- Filter bandwidth is the width of the frequency range over which signals are attenuated less than 3 dB – half the input power to the filter.

- Feed line loss is specified in dB per some length (100 feet or 100 meters is common) at a particular frequency.
- Antenna gain is given in dB, usually compared to an isotropic or dipole antenna.
- Power amplifier and preamplifier gain is usually given in dB.

How to Calculate Decibels

Decibels are calculated using the following formulas:

$$dB = 10\log\left(\frac{power}{reference\ power}\right)$$

$$dB = 20\log\left(\frac{voltage}{reference\ voltage}\right) = 20\log\left(\frac{current}{reference\ current}\right)$$

The "log" of a number is short for "logarithm" and is the answer to the question, "To what value does the logarithm's base value need to be raised in order to equal the number in question?" When calculating decibels, we use the *common logarithm* which uses a base value of 10. (The *natural logarithm*, usually abbreviated *ln*, uses a base value of *e*, which is 2.71828.) If the number in question is 100, the base value of 10 would have to be raised to the power of 2 to equal 100. i.e. $10^2 = 100$. Thus, the common logarithm of 100 is 2. Similarly, *log*(1000) = 3, *log*(1/10) = -1, and so forth. For all decibel calculations, use the common logarithm.

Why is the logarithm of voltage and current ratios multiplied by 20 instead of 10? First, decibels are always about power ratios, so don't think there is a "voltage dB" and a "current dB" that is different from a "power dB." A dB is a dB is a dB. Using the equations $P = V^2/R$ and $P = I^2R$ to substitute for the power values, you'll see that the ratios inside the parentheses of the decibel equation become V^2/V_{ref}^2 and I^2 / I_{ref}^2. Logarithms treat exponents specially: *log* (value^{Exp}) = Exp × *log* (value). So, in the case of the voltage and current ratios, the exponent of 2 is brought outside the logarithm calculation as 10 × 2 *log* (ratio) = 20 *log* (ratio).

Another useful characteristic of decibels is that gains and losses of stages in a radio system can be added together if they are specified in dB. For example, if you have an antenna with 8 dB of gain connected to a preamplifier with 15 dB of gain, the total gain is simply 8 + 15 = 23 dB. Similarly, if a power amplifier with 12 dB of gain is connected to a feed line with 1 dB of loss and then to an antenna with 4 dB of gain, the total gain of that combination is 12 – 1 + 4 = 15 dB. Losses are treated as negative gains.

Using a Calculator with Decibels

You will need a calculator that includes the *log* and the 10^X function to work with decibel values. (The 10^X function is sometimes labeled as log^{-1} or accessed with the *Inv* key followed by *log*. Read your calculator's manual if you are not clear about how to use these functions). Be sure that your calculator is set to calculate common logarithms and not natural logs.

Here are step-by-step instructions to use the scientific calculator that comes with the *Windows* operating system to calculate the ratio of 20 watts to 10 watts in decibels:

Step 1: If necessary, click C to clear the calculator, then enter 20.
Step 2: Click / to start the division, then enter 10, and click =. The display will show a value of 2.
Step 3: Click log. The display will show a value of 0.301…
Step 4: Click *, then enter 10, and click =. The display will show a value of 3.01…This is the value of the ratio 20/10 = 2 in dB.

To convert a decibel value to a ratio, use the following formulas:

$$Power\ ratio = 10^{(dB/10)}$$

$$Voltage\ or\ current\ ratio = 10^{(dB/20)}$$

To convert the value of 3 dB back to a power ratio, follow these steps:

Step 1: Enter 3, then click /, enter 10, and click =. The display will show a value of 0.3.
Step 2: Click 10^x. The display will show a value of 1.995… This is the value of the ratio with a decibel value of 3.

There are also many online converters that calculate decibels. Crown Audio offers one that uses a power ratio (**www.crownaudio.com/db-power.htm**) and one that uses a voltage ratio (**www.crownaudio.com/db-volts.htm**).

Decibel Shortcuts

You don't necessarily need to carry a calculator around with you all the time to work with decibels. You'll find that most of the time you can estimate the dB equivalent of a ratio or the ratio represented by a value in dB. Remembering a few values corresponding to common ratios and some powers of ten from the table of common decibel values will satisfy many ham radio needs!

Table of Common Decibel Values

Power ratio	*Decibel value (dB)*	*Voltage ratio*	*Decibel value (dB)*
0.001	-30	0.001	-60
0.01	-20	0.01	-40
0.1	-10	0.1	-20
0.125	-9	0.125	-18
0.25	-6	0.25	-12
0.5	-3	0.5	-6
0.79	-1	0.707	-1.5
1	0	1	0
1.26	1	1.414	1.5
2	3	2	6
4	6	4	12
5	7	5	14
8	9	8	18
20	13	20	26
50	17	50	34
10	10	10	20
100	20	100	40
1000	30	1000	60

Decibel values for ratios not in this table can often be calculated by using the property (a x b) in dB = (a) in dB + (b) in dB. Here are some examples:

- dB value of 25 = dB value of (5 x 5) = dB value of 5 + dB value of 5 = 7 + 7 = 14 dB
- dB value of 40 = dB value of (20 x 2) = dB value of 20 + dB value of 2 = 13 + 3 = 16 dB
- dB value of 0.2 = dB value of (0.1 x 2) = dB value of 0.1 + dB value of 2 = -10 + 3 = -7 dB
- dB value of 0.005 = dB value of (0.01 x 0.5) = dB value of 0.01 + dB value of 0.5 = -20 + (-3) = -23 dB

Special Decibel Abbreviations

You will often see the abbreviation dB followed by a letter. That means the value was calculated using a specific reference value. The letter indicates that the value is "decibels with respect to…" followed by the reference value. For example, you will frequently see power levels given in dBm. The lower case "m" stands for milliwatt (mW), with 0 dBm corresponding to the reference power of 1 mW. 10 dBm would be 10 times that or 10 mW. -6 dBm would be ¼ mW. In other words, dBm is another way of referring to power. It can make life a bit easier if you're doing system calculations. There are a number of other common abbreviations that specify certain reference levels and several are listed in the table.

Table of Decibel Reference Abbreviations

Abbreviation	*Reference value*
dBm	one milliwatt (1 mW)
dBW	one watt (1 W)
dBV	one volt (1 V)
dBμV	one microvolt (1 μV)
dBi	gain of an isotropic antenna
dBd	maximum gain of a half-wave dipole in free space
dBFS	full-scale value
dBc	carrier power

from **en.wikipedia.org/wiki/Decibel**

An example helps explain how dBm used. Say we have a transmitter that puts out 100 W, a feed line that has 3 dB loss and an antenna gain of 6 dB. Instead of having to use the decibel formula three times, convert the power to dBm once, do the additions and subtractions and convert back: 100 W = 100,000 mW = 10^5 mW, so the power level is

+50 dBm. Then we lose 3 dB in the coax, so we are down to +47 dBm (+50 minus the 3 dB loss). Finally, we gain 6 dB at the antenna for a net result of +53 dBm. We know this is 3 dB above 50 dBm, so the power is doubled to 200 W.

It is important to stress that while decibels represent change, dBm represents a particular power level. It's like saying, "I have so many watts of power." All the same rules of decibels apply when using dBm. How many dBm is 5 W of power? First, 1 W is 1000 mW or +30 dBm and 10 times that is 10 W, or +40 dBm. Half of that is +40 dBm – 3 dB or +37 dBm. Thus 5 W equals +37 dBm. You can find online calculators that convert between dBm and watts, such as the one at **www.radius.net/power-to-dbm-conversion.html**.

Decibels and Power Examples

Let's suppose you have an amateur transmitter that operates on the 2 meter band. Your transmitter has an output power of 10 watts, but you would like a little more power to use to make contact with a distant station. An amplifier is just what you need. After connecting your new amplifier, you measure the output power again, and find it is now 100 watts. How many dB increase is this? We'll use the 10-W signal as the reference in this case. Divide 100 W by 10 W to find the power ratio.

$$\text{Power Ratio} = \frac{P_1}{P_0} \qquad \text{(Equation 1)}$$

where

P_0 is the reference power level

P_1 is the power level compared to the reference power

$$\text{Power Ratio} = \frac{P_1}{P_0} = \frac{100\ \text{W}}{10\ \text{W}} = 10$$

Now find the logarithm of the power ratio.

$\log (10) = \log (10^1) = 1$

Finally, multiply this result by 10

decibels = $10 \times 1 = 10$ dB

Your amplifier has increased the power of your 2 meter signal by 10 dB!

Now suppose the amplifier increased your signal to 1000 watts. Choose the reference power to be 10 W again, and divide the new power by the reference.

$$\text{Power Ratio} = \frac{P_1}{P_0}$$

$$\text{Power Ratio} = \frac{P_1}{P_0} = \frac{1000\ \text{W}}{10\ \text{W}} = 100$$

Find the logarithm of the power ratio.

$\log (100) = \log (10^2) = 2$

Multiply this result by 10 to find the number of decibels.

decibels = $10 \times 2 = 20$ dB

If we put all these steps together into a single equation, we once again have the definition of a decibel.

$$\text{decibels (dB)} = 10 \log \left(\frac{P_1}{P_0} \right) \qquad \text{(Equation 2)}$$

where

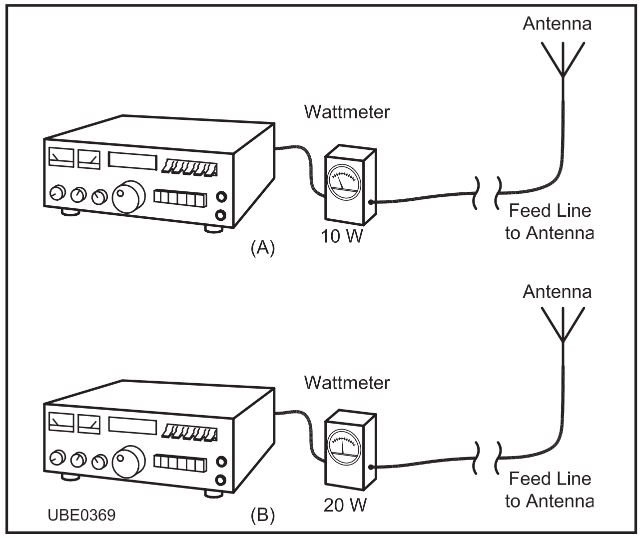
P_0 is the reference power level

P_1 is the power level compared to the reference power

Use this equation to calculate the number of decibels between power levels.

You should be aware of certain power ratios, because they occur so often. For example, let's see what happens if we double a given power. Suppose we start with a circuit that has a power of 2 mW. What dB increase occurs if we double the power to 4 mW? We'll start with the basic definition of a decibel.

Figure 1 — The output power from an Amateur Radio transmitter is 10 watts. After making some adjustments to the transmitter tuning, you measure the power again. Now you find the power has increased to 20 watts. The text describes how to calculate the decibel increase that occurred.



$$dB = 10 \log \left(\frac{P_1}{P_0}\right)$$

$$dB = 10 \log \left(\frac{4 \text{ mW}}{2 \text{ mW}}\right)$$

$$= 10 \log (2) = 10 \times 0.3 = 3.0 \text{ dB}$$

When we double the power, there is a 3 dB increase. This is true no matter what the actual power levels are. Let's look at an example with higher power levels to show that the dB increase is the same.

We measure the transmitter output power at an Amateur Radio station like the one shown in **Figure 1**, and find that it is 10 W. Use this power as a reference power for the station. After making some adjustments to the circuit, we measure the transmitter output power again. This time we find that the output power has increased to 20 W. What is the power increase, in dB? Equation 2 will help us answer this question.

$$dB = 10 \log \left(\frac{P_1}{P_0}\right)$$

$$dB = 10 \log \left(\frac{20 \text{ W}}{10 \text{ W}}\right)$$

$$= 10 \log (2) = 10 \times 0.3 = 3.0 \text{ dB}$$

So our transmitter adjustments gave us a 3 dB increase in transmitter power.

Suppose you measured the power output from another transmitter, and found it to be 100 W. Later, after experimenting with a new circuit in the transmitter, you measure the output power as 50 W. What effect did your experiment have on the output power? What is the power change in decibels? Again, Equation 2 will help answer this question.

$$dB = 10 \log \left(\frac{P_1}{P_0}\right)$$

$$dB = 10 \log \left(\frac{50 \text{ W}}{100 \text{ W}}\right)$$

$$= 10 \log (0.50)$$

$$= 10 \times (-0.30) = -3.0 \text{ dB}$$

Although the power levels in our two examples were much different, we still had a 3 dB change. This is an important point about the decibel. It compares two power levels. The number of decibels depends on the ratio of those

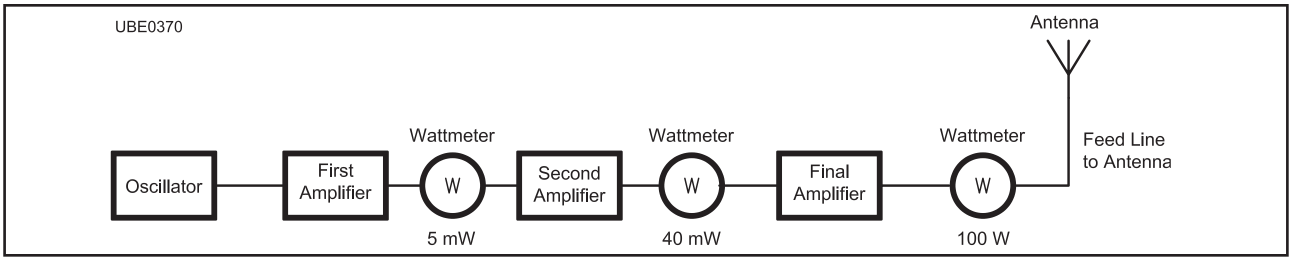


Figure 2 — A simple amateur transmitter amplifies the signal from an oscillator and then feeds that signal to an antenna. It uses several amplifier stages. The input power to one of those stages is 5 milliwatts and the output from that stage is 40 milliwatts. The text describes how to calculate the gain of that amplifier stage.

levels, not on the actual power. The 3 dB value is also important, because it shows that one power level was twice the other one. Increasing a power by two gives a 3 dB increase and cutting a power in half gives a 3 dB decrease.

Whenever you multiply or divide the reference power by a factor of 2, you will have a 3 dB change in power. You might guess, then, that if you multiplied the power by 4 it would be a 6 dB increase. If you multiplied the power by 8 it would be a 9 dB increase. You would be right in both cases!

Suppose the power in part of a circuit such as the one shown in **Figure 2** measures 5 milliwatts and in another part of the circuit it measures 40 mW. Using the 5-mW value as the reference power, how many decibels greater is the 40-mW power?

$$\text{dB} = 10 \log \left(\frac{P_1}{P_0} \right)$$

$$\text{dB} = 10 \log \left(\frac{40 \text{ mW}}{5 \text{ mW}} \right)$$

$$= 10 \log (8.0) = 10 \times 0.9 = 9.0 \text{ dB}$$

What happens if the power decreases? We can continue with the problem above, and measure the actual power arriving at the antenna. In this station, a long length of coaxial cable connects the transmitter to the antenna. Because some power is lost in this cable, we measure only 150 watts at the antenna. This time we'll use the 1500 W amplifier output as our reference. We want to compare the power at the antenna with the amplifier power. Again, Equation 1 helps us answer our question.

$$\text{dB} = 10 \log \left(\frac{P_1}{P_0} \right)$$

$$\text{dB} = 10 \log \left(\frac{150 \text{ W}}{1500 \text{ W}} \right)$$

$$\text{dB} = 10 \log (0.10) = 10 \log (10^{-1})$$

$$\text{dB} = 10 \times (-1) = -10 \text{ decibels}$$

The negative sign tells us that we have less power than our reference. Of course, we knew that because there was less power at the antenna than the amplifier was producing. What happened to that power? Some of the energy going through the coaxial cable changed to heat, and there may be other losses in the cable. All coaxial cables would have some loss. 10 dB of cable loss means that 90% of the power entering it is lost to heat, and that leaves only 10% at the cable output. If the loss were 20 dB, then 99% of the power entering it would be lost to heat, and that would leave only 1% at the cable output. Most cables do not have this much loss — the ratios are only used as examples for practicing calculations.

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Experiment #29: Kirchhoff's Laws

Who is this Kirchhoff guy and why are his laws so important? They form the basis of understanding circuits, even as simple rules of thumb. In this experiment, I'll introduce the two laws and show you how they're used—without a bar exam!

Terms to Learn

- *Branch*—a circuit path with two terminals through which current can flow
- *Node*—the junction of two or more branches
- *Loop*—any closed path through a circuit that visits nodes and branches only once

Introduction

Circuit analysis (an intimidating pair of words) is founded on Gustav Kirchhoff's Current and Voltage Laws, which he announced in 1845 as an extension of Georg Ohm's pioneering research. These two laws are consequences of the law of energy conservation. In an electronic circuit, just like anyplace else, electrical energy produced must be equal to energy consumed.

To understand the laws, it is important to use the right terms to describe a circuit. First, a *branch* is a circuit path with two terminals through which current can flow—a wire, a resistor, a coil or a box containing some arbitrary circuit. A *node* occurs where more than one branch comes together. A *loop* is a complete path through a circuit, beginning and ending at the same node, but not visiting a node or branch more than once.

Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law is the easiest to understand and it is applied at nodes, where currents combine, as shown in Figure 1. Even the simple connection between R3 and R4 is a two-branch node. (Don't confuse schematic connection "dots" with nodes because there may be more than one dot for a single node as shown at the bottom of the figure.)

KCL says that the sum of currents entering and leaving a node must equal zero. That seems reasonable, since electrons don't pile up at a circuit junction! KCL is a way of stating that energy must be conserved or balanced. The energy it takes to push currents through circuit branches into a node must equal the energy consumed in the branches through which the currents flow out of the node.

As an equation, KCL can be written as *incoming currents = outgoing currents* or *incoming currents – outgoing currents = zero*. You can assign a positive value to either incoming or outgoing current, so that currents flowing into and out of the node have opposite signs. Current is the same everywhere in the branch—you can't reverse current from one end of a branch to the other or change its value.

An example will help. Figure 2 shows a simple circuit with an arbitrary current assigned in each of the five branches. I1 through I5 are called *branch currents*. The three nodes are labeled 1, 2, and 3. We don't know which way the actual branch currents flow because we don't know whether V is positive or negative. The assigned direction doesn't matter! If we draw the arrow in the wrong direction, the calculated value for the branch current turns out to be negative.

Let's "do a KCL" for all three nodes. Ignore the green loop markings for now. At node 1, I1 is assumed to flow in and I2 and I4 to flow out. At node 2, I4 flows in and I3 and I5 flow out. At node 3, I2, I3, and I5 flow in and I1 flows out. If we decide that current flowing into a node is positive:

Node 1: I1 = I2 + I4
Node 2: I4 = I3 + I5
Node 3: I2 + I3 + I5 = I1

KCL is used when analyzing parallel connections in a circuit, such as when figuring out how current divides between two unequal resistances or determining the effect of combining currents.

Kirchhoff's Voltage Law (KVL)

KVL is also a consequence of the law of conservation of

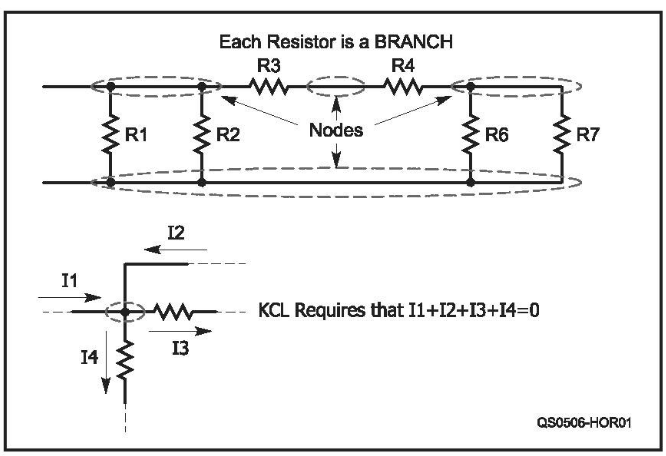

Figure 1—KCL requires the sum of currents at a node equal zero.

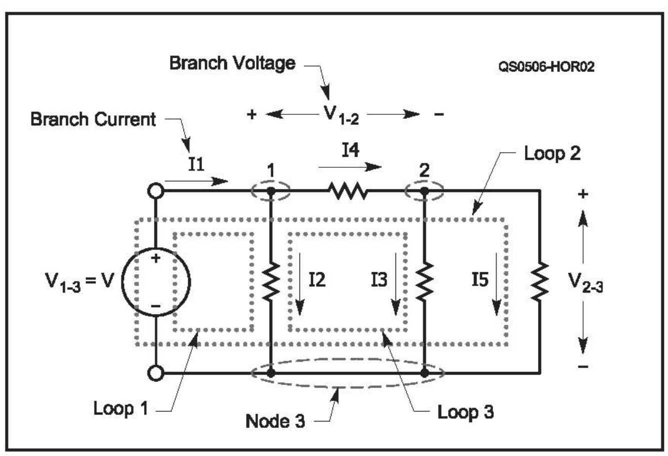

Figure 2—This circuit illustrates nodes, branches and loops—the keys to analyzing circuits.

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energy. The opposite of KCL, KVL is applied to loops and states that *the sum of branch voltages around a loop is equal to zero*. A *branch voltage* is the voltage from one end of the branch to the other, such as V1-2 between nodes 1 and 2 in Figure 2. For example, if we follow loop 1 from the power supply's positive terminal at node 1, through the branch for I2, and back to the supply's negative terminal at node 3, the voltages must all sum to zero. It would be the same if, instead, we followed loop 2 through I4, then I5, and back to the negative terminal.

Why does this conserve energy? Take the perspective of a single electron leaving the positive terminal of the power supply that gives the electron all of its energy. If the electron chooses to follow branch current I2, all of its energy is dissipated by that resistor before returning to the supply. It might also follow loop 2 and spend its energy in those resistors. In either case, the energy imparted by the supply has to be sufficient for the electron to "make it home." If the electron didn't expend all of its energy, it would arrive home with energy to spare, increasing the energy stored in the supply! KVL describes how energy is exactly balanced between *sources* (that supply energy) and *sinks* (that consume or dissipate energy).

As with KCL, you must keep polarities straight. By convention, voltages across an energy sink (such as a resistor) are assumed to be positive in the direction of the current—voltage is plus to minus across a resistor in the direction of current flow. Voltages through an energy source (such as a power supply) are negative in the direction of the current. Just as for KCL, if you don't know a voltage's polarity, you're allowed to guess and, if you're wrong, it turns out to be negative.

Let's do another example. In Figure 2, "doing a KVL" around loops 1, 2, and 3, the equations are:

Loop 1: $I2 \times R2 - V = 0$ or $I2 \times R2 = V$

Loop 2: $I4 \times R4 + I5 \times R5 - V = 0$ or $I4 \times R4 + I5 \times R5 = V$

We can move the energy sources to the other side of the equal sign and treat them as positive quantities, which is a little more convenient.

Loop 3: $I4 \times R4 + I3 \times R3 - I2 \times R2 = 0$

Note that there is no energy (voltage) source in loop 3. Furthermore, we encounter the voltage across R2 as negative to positive because of the assigned direction of I2. Bonus—there are three more possible loops in the circuit. Can you find them?

KVL is used when analyzing (or troubleshooting!) circuits using their voltages. For example, when looking at the collector circuit of a common-emitter amplifier, the resulting KVL equation balancing energy sources and sinks is $V_{CC} = I_C R_C + V_{CE} + I_E R_E$.

Extending the Laws to AC Circuits

KCL and KVL work just as well when resistance is replaced by impedance, which includes both resistance and reactance. Impedance generally changes with frequency, so the equations for circuit voltages and currents will also depend on frequency.

For example, if a resistor and capacitor are connected in parallel, KCL will show that at dc, all the current goes through the resistor, gradually shifting to the capacitor as frequency increases. In the series connection of a resistor and inductor, KVL will show that the voltage across the resistor is a maximum at dc and gradually drops as frequency increases.

Exercising Kirchhoff's Laws

Now test KCL and KVL in a real circuit! The solutions for this circuit are found on the Hands-On Radio Web page: **www.arrl.org/tis/info/HTML/Hands-On-Radio**.

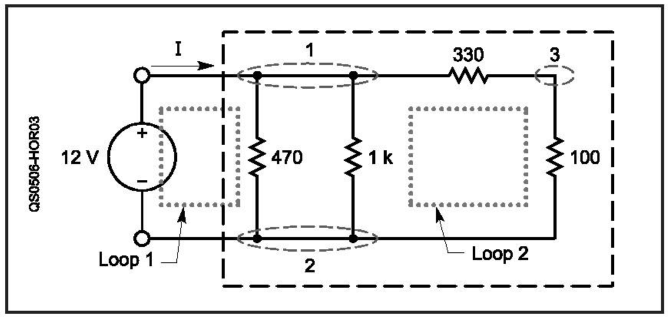

Figure 3—Build this circuit to test your understanding of KCL and KVL.

- Using the circuit of Figure 3, combine the values of the series and parallel resistors until you have one single equivalent resistance, R_{EQ} replacing everything inside the dashed line.
- Solve for $I = V / R_{EQ} = 12\ V / R_{EQ}$.
- What current flows in the 470 Ω and 1 kΩ resistors with 12 V across them?
- What current flows through the series combination of the 330 Ω and 100 Ω resistors?
- Build the circuit of resistors (no power supply yet) on your prototype board and measure the resistance from node 1 to node 2 to see if your calculated value of R_{EQ} is correct.
- Apply 12 V as shown and measure the power supply current, I. Compare the value to your calculated value.
- Measure all of the currents going into and out of the three nodes and confirm that KCL works. Either measure the currents directly, using the current scale of your meter, or indirectly, by measuring voltage across the resistors and using Ohm's Law.
- Measure all of the voltages in the two loops and confirm that KVL works. Don't forget to always measure voltage in the same "direction" around the loop.
- Experiment by changing the resistor values, then doing the calculations and measurements again. Identify the two remaining loops and "do a KVL" around them. Try replacing the 330 Ω resistor with a diode!

Shopping List

- 100 Ω, 330 Ω, 470 Ω, and 1 kΩ 1/4 W resistors

Suggested Reading

The section on series and parallel resistances in Chapter 4 of *The ARRL Handbook* (2005) covers Kirchhoff's Laws and also has all the equations for combining series and parallel resistances if you're a little rusty on those. While you're at it, browse through the following section on Thevenin equivalents—we'll be tackling those in the future. Rick, KB1HUE, also contributes the following Web site reference, which, if you have a Macintosh (or Mac simulator software), will provide hours of fun: **www.inform.umd.edu/EdRes/Topic/Chemistry/ChemConference/Software/ElectroSim/index.html**.

Next Month

We'll learn about another special type of IC—the charge pump. These handy critters can turn positive into negative or even double a voltage, just with a clever arrangement of switches and a couple of capacitors.

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Experiment 117

Laying Down the Laws

Most amateurs tend to think of *wireless* as beginning with Marconi in the mid-1890s — he transmitted a message over a distance of a bit less than 2 miles in 1895. The historically minded ham might travel farther back through the experiments and papers of well known and not so well known names such as Nikola Tesla, Heinrich Hertz, Nathan Stubblefield and Mahlon Loomis to arrive at James Clerk Maxwell's electromagnetic theory, published in 1864. Yet their work required, as Isaac Newton had characterized it, "standing upon the shoulders of giants" who explored the *terra incognita* of electricity and magnetism from the early 1600s. Who were these giants and what did they discover?

My interest in prewireless was sparked, so to speak, by a recent article in the *IEEE Antennas and Propagation Magazine* giving a chronology of how wireless communications came to be.[1] (The article may be available through your local library or from an *IEEE* member.) A huge number of discoveries and explanations of basic concepts were required before Maxwell could synthesize them into his theory of electromagnetic waves.

For many of us, our electrical education began with Ohm's law, first stated by Georg Ohm in 1827.[2] We know it today as the familiar $I = E / R$, but R was a brand new idea in those days. In fact, Ohm's ideas were not well received at all! From Ohm's law, we progressed through the equation for power ($P = E \times I$) and then into circuitry such as capacitance, inductance, reactance and impedance that quickly followed. But capacitance and inductance are treated as *givens* in equations we memorize for time constants, turns ratio, resonant frequency and so forth. Where do these *proto wireless* concepts come from?

In this column, we'll begin reviewing several of the most important advances listed in the article then progress to some simple experiments you can do yourself. The goal is to more fully understand what is meant by the familiar symbols and units in the design equations and in schematics. It is one thing to memorize an equation or paragraph and quite another to experience it for yourself on the workbench!

[1]Salazar-Palma, *et al*, "The Father of Radio: A Brief Chronology of the Origin and Development of Wireless Communications," *IEEE Antennas and Propagation Magazine*, Vol 53, No 6, Dec 2011, pp 83-114.

[2]en.wikipedia.org/wiki/Ohm's_law

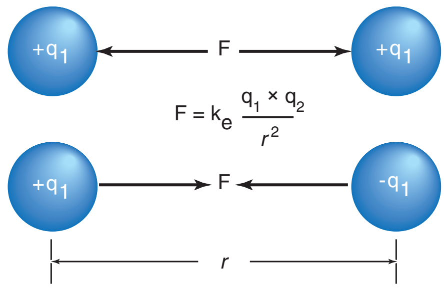

Figure 1 — Coulomb's law describes the force, F, between two electrically charged particles (q_1 and q_2). The force is proportionally weaker with the square of the distance between the particles, r. If the charges have the same polarity, the force is positive and they repel each other. If they are oppositely charged, the force is negative and attracts the particles together.

The Beginnings

There is a long history of experimentation with static electricity and magnetism leading to the invention of the capacitor in the mid 1700s. Perhaps the best known example of an early capacitor is the Leyden jar.[3] Since static electricity was fairly easy to generate, the capacitor and its ability to store electrical energy were well known by the end of the 18th century. The relationship between electricity and magnetism, however, was quite unclear and that relationship lies at the root of electromagnetic phenomena — such as wireless.

The IEEE article begins its journey to wireless with Charles-Augustin Coulomb's determination in 1785 that electric forces varied proportionately to the inverse square of distance — now known as Coulomb's law (Equation 1) as illustrated in Figure 1.

[3]en.wikipedia.org/wiki/Leyden_jar

$$F = k_e \frac{q_1 q_2}{r^2} \quad \text{[Eq 1]}$$

where

F = the electric force between two particles with charges, q_1 and q_2,

r = the distance between them, and

k_e = a "constant of proportionality." It is this constant that turned out to have the most for reaching implications because it is determined solely by the properties of free space:

$$k_e = \frac{1}{4\pi\varepsilon_0} = \frac{c^2\mu_0}{4\pi} \quad \text{[Eq 2]}$$

where

c = speed of light in vacuum

ε_0 = the permittivity of free space (roughly, the ability of free space to contain electrical energy) and

μ_0 = the permeability of free space (the ability of free space to contain magnetic energy).

As it turns out, the speed of light (electromagnetic energy) traveling in free space is also determined by these two quantities:

$$c = \sqrt{\frac{1}{\varepsilon_0\mu_0}} \quad \text{[Eq 3]}$$

Not only does Coulomb's simple relationship contain the beginnings of wireless but it is also the first step in the studies of electromagnetic waves that led to relativity and its profound effects on our understanding of the universe. Coulomb did not know this at the time, of course. He only knew that he had discovered a relationship between electrical charge and electrical force.

Meanwhile (as the narrator often intones) other investigators were developing new ways of creating electricity. Up until this time, electrical experiments had to be performed with static electricity created by mechanical friction. In 1799, Alessandro Volta created an electrochemical *battery* based on chemical principles.[4] This was a major advance because experimenters then had a source not only of what Volta called the *electromotive force* (abbreviated *EMF*) but a source of

[4]en.wikipedia.org/wiki/History_of_the_battery

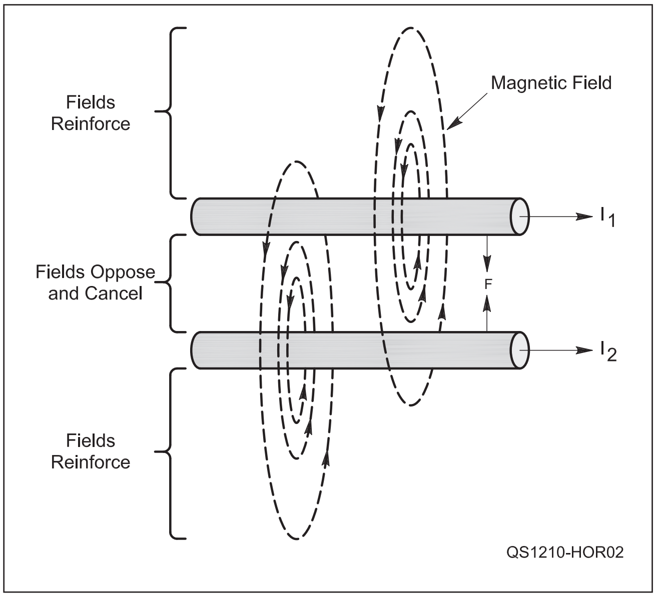

Figure 2 — Ampère's force law describes the force between two parallel, current-carrying wires. If the currents are flowing in the same direction, the fields are oriented in opposite directions between the wires, partially cancelling each other. Since the fields reinforce elsewhere, the result is a force pushing the wires together.

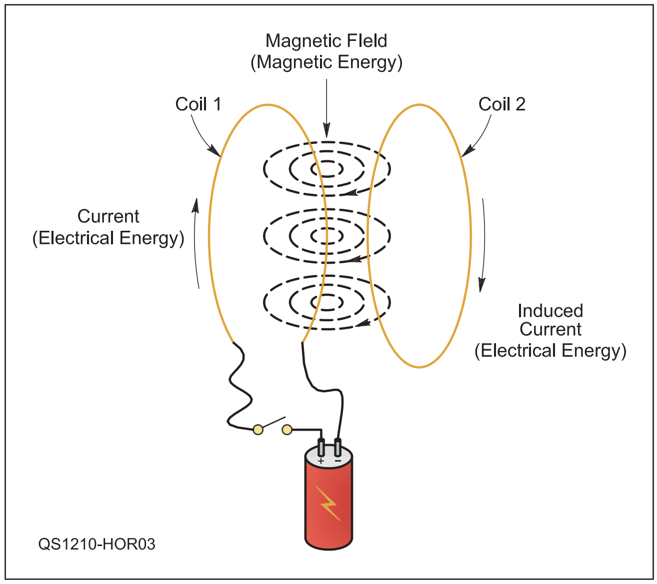

Figure 3 — Faraday demonstrated electromagnetic induction by showing how changing current in one coil induces a similar current in a second coil through a shared magnetic field. When the switch is closed, current in coil 1 will cause the current shown in coil 2. Lenz's law states that the current in coil 2 will be oriented to oppose the magnetic field from coil 1.

current they could then control and study. Prior to that current was mostly available as pulses from electrical discharges — *sparks*.

Magnetism was considered a separate phenomenon from electricity until 1820 when Hans Christian Ørsted discovered that current flowing through a wire caused a magnetic compass needle to deflect and created a circular magnetic field around the wire.[5] François Arago then demonstrated that not only did current flowing through a wire affect a magnet but that the current carrying wire itself became a magnet! Within days, André-Marie Ampère also demonstrated that parallel currents attract each other and opposing currents repel due to those magnetic fields:

$$F = 2k_a \frac{I_1 I_2}{r} \qquad \text{[Eq 4]}$$

where $k_a - k_e/4\pi$,

with similar definitions to Coulomb's law and illustrated by Figure 2. Note the similarity of Ampère's force law and Coulomb's law above.

The linkage of electricity and magnetism through the motion of electrical charge — current — led Ampère to create a theory of *electrodynamics* that is at the heart of wireless. After all, it is the continual acceleration and deceleration of electrons in our antennas that cause electromagnetic waves to be radiated. The movement of electrons in response to incoming waves allows us to hear those waves in our receivers. 1820 was a very good year!

In 1825 and 1826, Ampère published a collection of material on magnetism including what is now known as Ampère's law, the general relationship between currents and magnetic fields. This relationship was extended by Maxwell and forms one of Maxwell's equations that describe electromagnetic fields.

Getting Ready for Maxwell

Almost immediately, Ørsted's discovery and Arago's extension of it led to practical inventions. In 1821, American physicist Joseph Henry invented the *electromagnet* by winding the current carrying wire into a coil. While doing these experiments, he also discovered the need for insulation between the wires making up the coil. His experiments led to refinement of the electromagnet into the *electromagnetic telegraph* in 1831.

The really big news of that year, however, came from Michael Faraday, a self taught scientist who had been experimenting with electricity and chemistry since 1812. Faraday demonstrated *electromagnetic induction* by showing how changing currents in one circuit (later *ac current*) could induce similarly changing currents in another circuit without any direct connection between them.[6]

Figure 3 shows that in doing so, Faraday converted the electrical energy of current in the first circuit into magnetic energy in the surrounding field and back into electrical energy in the second circuit. This led Faraday to predict the existence of electromagnetic waves, as well.

Faraday refined his explanation of induction into the following formula known as Faraday's law:

$$\varepsilon = -\frac{\text{change in } \Phi_B}{\text{change in time}} \qquad \text{[Eq 5]}$$

where ε is the electromotive force (or *EMF*) and the fraction represents the change in magnetic flux (Φ_B) with time. The faster the magnetic flux changes or the larger the amount of change in one circuit, the larger the voltage that is *induced* in the other circuit.

The minus sign in the equation means that the current caused by the changing magnetic field flows in the direction that creates an *opposing* magnetic field. This is otherwise known as Lenz's law and it describes the *back EMF* we observe in motors and the *kickback voltage* in a relay coil when the relay is deenergized. If you look closely at Figure 3, you can see that the induced current flows in the opposite direction to the current caused by the battery.

In the next experiment, we will follow in the steps of Coulomb, Ampère, Ørsted and Faraday by performing some simple experiments that demonstrate the various effects they described. Is this purely a historical exercise? Not at all! These phenomena are at the heart of every radio — without them we would be wireless less.

[5]**en.wikipedia.org/wiki/Hans_Christian_%C3%98rsted**

[6]**en.wikipedia.org/wiki/Electromagnetic_induction**

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Experiment 118

The Laws at Work

Last month's column ended in the year 1831 as Michael Faraday discovered *electromagnetic induction*. In fact, the day on which I finished this column was the 181st anniversary of that discovery — August 29th.

As a review, Faraday showed how changing currents in one circuit could induce similarly changing currents in another circuit without any direct connection between them. This occurs by first converting the first circuit's electrical energy into magnetic energy in the surrounding field and then back into electrical energy in the second circuit. Electromagnetic induction is described by Faraday's Law as explained in last month's column.[1] The faster the magnetic flux changes because of motion or changes in current, the larger the voltage that is induced in the other circuit.

The minus sign in Faraday's law gives rise to Lenz's law: The *electromotive force* (EMF) induced by the changing magnetic field causes current to flow in the direction that creates a magnetic field *opposing* the original change in magnetic flux. Lenz's law describes the *back EMF* we observe in motors and the *kickback voltage* in a relay coil when it is deenergized. Last month's Figure 3 shows the relative direction of the currents in both circuits.

There are several ways to cause the magnetic field linking the two coils to change. The most obvious way is to simply turn the current generating the field on and off with a switch. Another way is to move one of the coils so it encounters more or less of the magnetic field. This is the principal behind an electrical ac generator. An *armature* coil is rotated inside a current-carrying stationary coil (or *stator*) causing the armature voltage to vary as a sine wave. Similarly, changing the current in the stator causes the armature to turn, creating a motor.

In this month's column we will follow in the steps of pioneers André-Marie Ampère, Hans Christian Ørsted and Faraday by performing some simple experiments demonstrating the various effects they described. Is this purely a historical exercise? Not at all! These phenomena are at the heart of radio — without them we would be wireless-less.

Experiment #1 — Ørsted's Observation

During a lecture, with experimental apparatus scattered across a table, Ørsted noticed that a compass needle deflected away from north when he switched current on and off in a nearby circuit. This was the first observation linking electricity and magnetism and it was the proverbial *Big Deal* in 1820. So let's repeat it.

Head for the kitchen and "borrow" a strong refrigerator magnet capable of holding up a calendar. You'll also need a source of dc power such as a power supply — 12 V is fine — and a coil of several mH with a dc resistance of 10 to 30 Ω. You can either buy or scrounge a suitable inductor but the coil of a 12 V relay will do fine and you may have one in your junk box. (The relay used in experiments #107 through #110 is a good choice. Alternately, a RadioShack 275-001 will work — remove the plastic case to see the coil.)

Working on a nonferrous table (plastic or wood), suspend the magnet a few inches above the table using thread or dental floss. Mark one face with pencil or tape. Connect the inductor to the 12 V supply through a voltmeter configured to measure current — start with the scale for measuring several amps and use a more sensitive scale after confirming the current is low enough not to

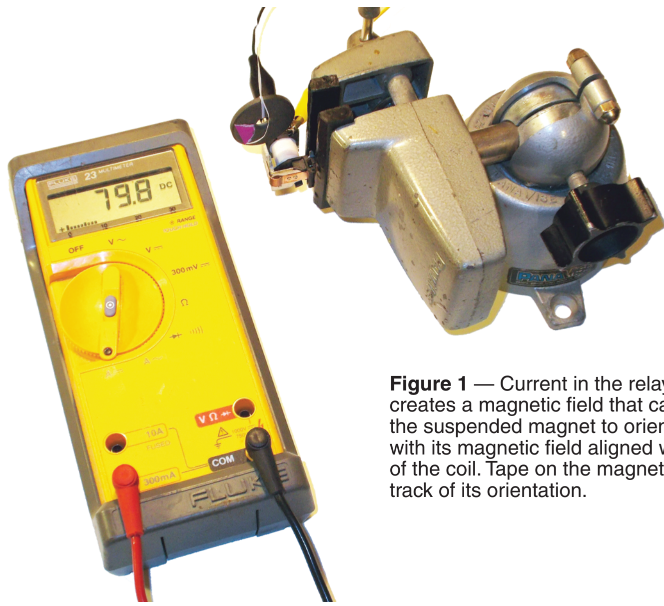

Figure 1 — Current in the relay coil creates a magnetic field that causes the suspended magnet to orient itself with its magnetic field aligned with that of the coil. Tape on the magnet keeps track of its orientation.

When Is E a V and V an E?

Beginners in electronics are often confused about the interchange of V and E in equations and formulas. The term "electromotive force" further muddies the water. When should each be used? Unfortunately, there is no standard definition or convention as described at **en.wikipedia.org/wiki/Electromotive_force**. Nevertheless, in ham radio E is usually used if referring to an electric field (such as the E field of an antenna) or if an electric field causes some effect or action (such as back EMF of an inductor). V is used to describe the difference in voltage between two points in a circuit, or the terminal voltage of a power supply or battery. V is also used as an abbreviation for volts, the unit of voltage.

[1]All previous Hands-On Radio experiments are available to ARRL members at **www.arrl.org/**

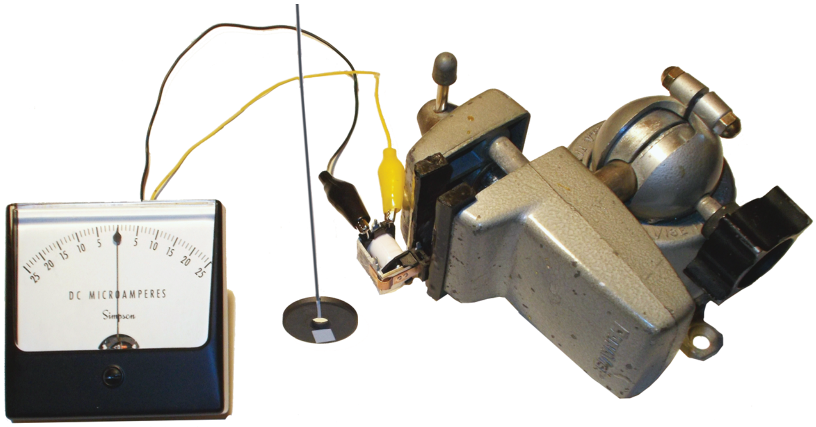

Figure 2 — Moving the magnet toward the coil changes the magnetic field in the coil and induces a current of one polarity in the circuit, causing the meter to deflect. Moving the magnet away from the coil causes an opposite change in the field and reverses the deflection of the meter.

Figure 3 — Current applied from a battery demonstrates the Ampère force separating the two parallel conductors. The conductors are made from a strip of aluminum foil. Wear protective eyewear and gloves when performing this experiment.

overload the meter. Place the coil near the magnet but in a position where it cannot contact the coil or the connecting wires. Wait for the magnet to reach complete rest.

Switch on the current while watching the magnet — the magnet will pivot and move, eventually stabilizing in a fixed position as in Figure 1. (My relay coil is drawing about 80 mA.) When you switch off the current, the magnet will return to its original position. The magnet is moving so that its magnetic field is aligned with the magnetic field of the energized coil.

Cycle the current on and off several times. The magnet will always stabilize with one face of the magnet in the same position. Slowly rotate the coil and verify that the magnet rotates to follow the orientation of the coil. Reverse the power supply leads and verify that the magnet stabilizes with the marked face reversed as well.

Experiment #2 — Faraday's Law

Technically, we're not going to demonstrate Faraday's law exactly so as to keep the setup simple. However, you will be able to see the effect of the orientation of the field on the direction of the induced current in a circuit. You will need a sensitive microammeter or millivoltmeter (also known as a *galvanometer*) capable of showing current or voltage of both polarities. Digital meters work okay, but if you have or can borrow a sensitive analog meter, the effect is easier to see. I found a mint condition 25-0-25 µA meter for $5 at a hamfest so keep your eyes open.

Figure 2 shows the basic setup. Disconnect your coil from the power supply and connect it directly between the meter terminals. Polarity is not important. (If you have a digital meter, use the most sensitive voltage scale, usually 200 or 300 mV.) Hold the magnet in your fingers and move it toward the coil and then away from the coil while watching the meter. You should see the meter deflect or indicate in one direction as the magnet moves toward the coil and then in the other direction as the magnet moves away. Experiment with different orientations of the magnet as you move it past the coil.

There are a number of great YouTube videos that demonstrate Lenz's law in which the induced current creates an opposing magnetic field.[2] You may have seen someone drop a magnet into a copper pipe through which it then falls very slowly. The induced currents are called *eddy currents* and they set up a magnetic field that is almost, but not quite, strong enough to stop the magnet. If the magnet did stop, there would be no eddy current and opposing field so, as a result, the magnet moves just fast enough to overcome the resistive losses in the pipe.

Experiment #3 — Ampère's Force Law

The final experiment is at the heart of what makes an electromagnetic rail gun work — the magnetic force between two current-carrying conductors. Figure 2 in the previous experiment shows how the magnetic fields of two parallel conductors carrying current in the same direction align to force the conductors towards each other. In this experiment, we'll show opposite currents forcing the conductors apart.

Head back to the kitchen and find the aluminum foil. Use a measuring stick to tear or cut off a strip about 18 to 24 inches long and about ¾ inches wide. Back in the lab, suspend the aluminum foil so that it forms a long, narrow U above the table as in Figure 3. Attach heavy clip leads to the open ends of the foil strip.

Get four fresh D cell batteries and use a battery holder or plastic tube to connect them in series — *do not* use a larger battery as the resulting higher currents may be unsafe. Wear eye protection and work gloves for the following step although it is unlikely that any significant heating will occur. Attach one clip lead to the battery's positive terminal cap. Tape or secure both clip leads so that moving the free ends does not cause the foil to move. When the foil is completely still, brush the remaining clip lead's free end against the battery's negative terminal momentarily — you'll see the foil loop expand slightly as several amperes of current flow through it, creating a force separating the parallel conductors. Experiment with different orientations of the foil or create two parallel strips that carry current in the same direction and are forced closer together.

Applying the Law

Do these experiments have any practical application in ham radio today? Most certainly! The effect observed by Ørsted is replicated in every analog meter movement. Faraday's and Lenz's laws are the foundation of transformers and shielded wires. If you've ever seen the inside of a high-voltage power supply after a short circuit, you've also seen the effects of Ampère's force law at work.

[2]YouTube videos **www.youtube.com/watch?v=kU6NSh7hr7Q** and **www.youtube.com/watch?v=G7ysnXH53Wo** are particularly good.

Radio Mathematics

Understanding radio and electronics beyond an intuitive or verbal level requires the use of some mathematics. None of the math is more advanced than trigonometry or advanced algebra, but if you don't use math regularly, it's quite easy to forget what you learned during your education. In fact, depending on your age and education, you may not have encountered some of these topics at all.

It is well beyond the scope of this book to be a math textbook, but this section touches some of topics that most need explanation for amateurs. For introductory-level tutorials and explanations of advanced topics, a list of on-line, no-cost tutorials is presented in the sidebar, "Online Math Resources". You can browse these tutorials whenever you need them in support of the information in this reference text.

Online Math Resources

The following Web links are a compilation of on-line resources organized by topic. Other resources are available online at **www.arrl.org/tech-prep-resource-library**. Look for the Math Tutorials section.

Many of the tutorials listed below are part of the *Interactive Mathematics* Web site (**www.intmath.com**), a free, online system of tutorials. The system begins with basic number concepts and progresses all the way through introductory calculus. The lessons referenced here are those of most use to a student of radio electronics.

Basic Numbers & Formulas

Order of Operations — **www.intmath.com/Numbers/3_Order-of-operations.php**
Powers, Roots, and Radicals — **www.intmath.com/Numbers/4_Powers-roots-radicals.php**
Introduction to Scientific Notation — **www.ieer.org/clssroom/scinote.html**
Scientific Notation — **www.intmath.com/Numbers/6_Scientific-notation.php**
Ratios and Proportions — **www.intmath.com/Numbers/7_Ratio-proportion.php**
Geometric Formulas — **www.equationsheet.com/sheets/Equations-4.html**
Tables of Conversion Factors — **en.wikipedia.org/wiki/Conversion_of_units**

Metric System

Metric System Overview — **en.wikipedia.org/wiki/Metric_system**
Metric System Tutorial —see Unit 3 at **www.arrl.org/radio-lab-handbook**

Conversion of Units

Metric-English — **www.m-w.com/mw/table/metricsy.htm**
Metric-English — **en.wikipedia.org/wiki/Metric_yardstick**
Conversion Factors — **oakroadsystems.com/math/convert.htm**

Fractions

Equivalent Fractions — **www.intmath.com/Factoring-fractions/5_Equivalent-fractions.php**
Multiplication and Division— **www.intmath.com/Factoring-fractions/6_Multiplication-division-fractions.php**
Adding and Subtracting — **www.intmath.com/Factoring-fractions/7_Addition-subtraction-fractions.php**
Equations Involving Fractions — **www.intmath.com/Factoring-fractions/8_Equations-involving-fractions.php**
Basic Algebra — **www.intmath.com/Basic-algebra/Basic-algebra-intro.php**

Graphs

Basic Graphs — **www.intmath.com/Functions-and-graphs/Functions-graphs-intro.php**
Polar Coordinates — **www.intmath.com/Plane-analytic-geometry/7_Polar-coordinates.php**
Exponents & Radicals — **www.intmath.com/Exponents-radicals/Exponent-radical.php**
Exponential & Logarithmic Functions — **www.intmath.com/Exponential-logarithmic-functions/Exponential-log-functionsintro.php**

Trigonometry

Basic Trig Functions — **www.intmath.com/Trigonometric-functions/Trig-functions-intro.php**
Graphs of Trig Functions — **www.intmath.com/Trigonometric-graphs/Trigo-graph-intro.php**

Complex Numbers

Complex Numbers — **www.intmath.com/Complex-numbers/imaginary-numbers-intro.php**

Working With Decibels

The *decibel* (*dB*) is a way of expressing a ratio *logarithmically*, meaning as a power of some *base* number, such as 10 or the base for *natural logarithms*, the number e ≈ 2.71828. A decibel, using the metric prefix "deci" (d) for one-tenth, is one-tenth of a *Bel* (B), a unit used in acoustics representing a ratio of 10, and named for the telephone's inventor, Alexander Graham Bell. As it turns out, the Bel was too large a ratio for common use and the deci-Bel, or decibel, became the standard unit.

In radio, the logarithmic ratio is used, but it compares signal strengths — power, voltage, or current. The decibel is more useful than a *linear* ratio because it can represent a wider scale of ratios. The numeric values encountered in radio span a very wide range and so the dB is more suited to discuss large ratios. For example, a typical receiver encounters signals that have powers differing by a factor of 100,000,000,000,000 (10^{14} or 100 trillion!). Expressed in dB, that range is 140, which is a lot easier to work with than the preceding number, even in scientific notation!

The formula for computing the decibel equivalent of a power ratio is

$$\text{dB} = 10 \log (\text{power ratio}) = 10 \log (P1/P2) \qquad (1)$$

or if voltage is used

$$\text{dB} = 20 \log (\text{voltage ratio}) = 20 \log (V1/V2) \qquad (2)$$

For equations 1 and 2 to produce equivalent results, both of the voltages must be measured across equivalent impedances. Otherwise, impedance must be accounted for according to $P = V^2 / Z$.

Positive values of dB mean the ratio is greater than 1 and negative values of dB indicate a ratio of less than 1. For example, if an amplifier turns a 5 W signal into a 25 W signal, that's a gain of 10 log (25 / 5) = 10 log (5) = 7 dB. On the other hand, if by adjusting a receiver's volume control the audio output signal voltage is reduced from 2 V to 0.1 V, that's a loss: 20 log (0.1 / 2) = 20 log (0.05) = –26 dB.

If you are comparing a measured power or voltage (P_M or V_M) to some reference power (P_{REF} or V_{REF}) the formulas are:

$$\text{dB} = 10 \log (P_M / P_{REF})$$

$dB = 20 \log (V_M / V_{REF})$

There are several commonly-used reference powers and voltages, such as 1 V or 1 mW. When a dB value uses one of them as the references, dB is followed with a letter. Here are the most common:

- dBV means dB with respect to 1 V (V_{REF} = 1 V)
- dBμV means dB with respect to 1 μV (V_{REF} = 1 μV)
- dBm means dB with respect to 1 mW (P_{REF} = 1 mW)

If you are given a ratio in dB and asked to calculate the power or voltage ratio, use the following formulas:

$$\text{Power ratio} = \text{antilog} (dB / 10) \quad (3)$$

$$\text{Voltage ratio} = \text{antilog} (dB / 20) \quad (4)$$

Example: A power ratio of –9 dB = antilog –(9 / 10) = antilog (–0.9) = 1/8 = 0.125

Example: A voltage ratio of 32 dB = antilog (32 / 20) = antilog (1.6) = 40

Antilog is also written as $\log^{-1}$ and may be labeled that way on calculators.

CONVERTING BETWEEN DB AND PERCENTAGE

You may also have to convert back and forth between decibels and percentages. Here are the required formulas:

$$dB = 10 \log (\text{percentage power} / 100\%) \quad (5)$$

$$dB = 20 \log (\text{percentage voltage} / 100\%) \quad (6)$$

$$\text{Percentage power} = 100\% \times \text{antilog} (dB / 10) \quad (7)$$

$$\text{Percentage voltage} = 100\% \times \text{antilog} (dB / 20) \quad (8)$$

Example: A power ratio of 20% = 10 log (20% / 100%) = 10 log (0.2) = –7 dB

Example: A voltage ratio of 150% = 20 log (150% / 100%) = 20 log (1.5) = 3.52 dB

Example: –1 dB represents a percentage power = 100% × antilog (–1 / 10) = 79%

Example: 4 dB represents a percentage voltage = 100% × antilog (4 / 20) = 158%

Computer Software and Calculators

Every version of the *Windows* operating system comes with a calculator program located in the Accessories program group and a number of free calculators are available on-line. Enter "online" and "calculator" into the search window of an Internet search engine for a list. If you can express your calculation as a mathematical expression, such as sin(45) or 10log(17.5), it can be entered directly into the search window at **www.google.com** and the Google calculator will attempt to solve for the answer.

Microsoft *Excel* (and similar spreadsheet programs) also make excellent calculators. If you are unfamiliar with the use of spreadsheets, here are some online tutorials and help programs:

homepage.cs.uri.edu/tutorials/csc101/pc/excel97/excel.html
www.baycongroup.com/el0.htm
www.bcschools.net/staff/ExcelHelp.htm

For assistance in converting units of measurement, the Web site **www.onlineconversion.com** is very useful. The Google online unit converter can also be used by entering the required conversion, such as "12 gauss in tesla", into the Google search window.

Decibels In Your Head

Every time you increase the power by a factor of 2 times, you have a 3-dB increase of power. Every 4 times increase of power is a 6-dB increase of power. When you increase the power by 10 times, you have a power increase of 10 dB. You can also use these same values for a decrease in power. Cut the power in half for a 3-dB loss of power. Reduce the power to 1/4 the original value for a 6-dB loss in power. If you reduce the power to 1/10 of the original value you will have a 10-dB loss. The power-loss values are often written as negative values: –3, –6 or –10 dB. The following tables show these common decibel values and ratios.

Common dB Values and Power Ratios

P2/P1	*dB*
0.1	–10
0.25	–6
0.5	–3
1	0
2	3
4	6
10	10

Common dB Values and Voltage Ratios

V2/V1	*dB*
0.1	–20
0.25	–12
0.5	–6
0.707	–3
1.0	0
1.414	3
2	6
4	12
10	20

You can also derive all the dB equivalents of integer ratios by adding or subtracting dB values. For example, to calculate the dB for a power ratio of 10/4 (2.5), subtract the dB equivalents for 10 and 4: 10 – 6 = 4 dB. Similarly for a ratio of 10/2 (5), 10 – 3 = 7 dB. The ratio of 5/4 (1.25) is 7 – 6 = 1 dB and so forth. The same trick can be used with the voltage ratios.

Rectangular and Polar Coordinates

Graphs are drawings of what equations describe with symbols — they're both saying the same thing. Graphs are used to present a visual representation of what an equation expresses. The way in which mathematical quantities are positioned on the graph is called the *coordinate system*. *Coordinate* is another name for the numeric scales that divide the graph into regular units. The location of every point on the graph is described by a pair of *coordinates*.

The two most common coordinate systems used in radio are the *rectangular-coordinate* system shown in **Figure 1** (sometimes called *Cartesian coordinates*) and the *polar-coordinate* system shown in **Figure 2**.

The line that runs horizontally through the

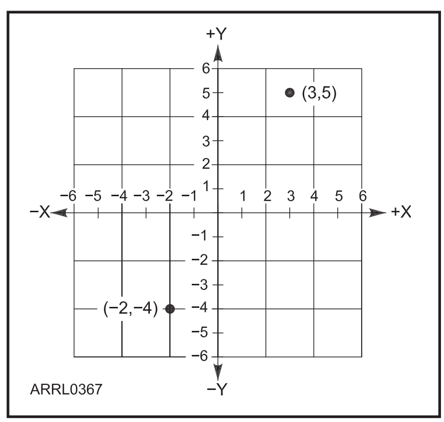

Figure 1 — Rectangular-coordinate graphs use a pair of axes at right angles to each other; each calibrate in numeric units. Any point on the resulting grid can be expressed in terms of its horizontal (X) and vertical (Y) values, called coordinates.

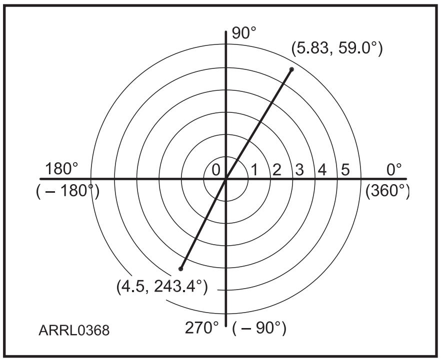

Figure 2 — Polar-coordinate graphs use a radius from the origin and an angle from the 0º axis to specify the location of a point. Thus, the location of any point can be specified in terms of a radius and an angle.

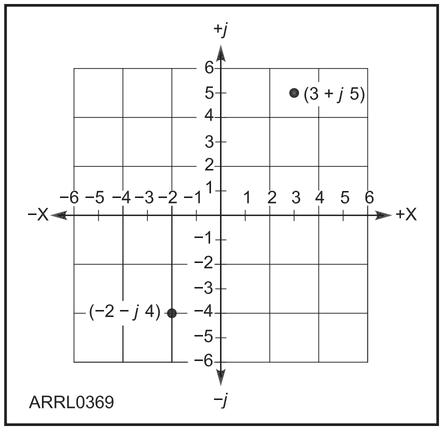

Figure 3 — The Y axis of a complex-coordinate graph represents the imaginary portion of complex numbers. This graph shows the same numbers as in Figure 1, graphed as complex numbers.

center of a rectangular coordinate graph is called the *X axis*. The line that runs vertically through the center of the graph is normally called the *Y axis*. Every point on a rectangular coordinate graph has two coordinates that identify its location, X and Y, also written as (X,Y). Every different pair of coordinate values describes a different point on the graph. The point at which the two axes cross — and where the numeric values on both axes are zero — is called the *origin*, written as (0,0).

In Figure 1, the point with coordinates (3,5) is located 3 units from the origin along the X axis and 5 units from the origin along the Y axis. Another point at (–2,–4) is found 2 units to the left of the origin along the X axis and 4 units below the origin along the Y axis. Do not confuse the X coordinate with the X representing reactance! Reactance is usually plotted on the Y-axis, creating occasional confusion.

In the polar-coordinate system, points on the graph are described by a pair of numeric values called *polar coordinates*. In this case, a length, or *radius*, is measured from the origin, and an *angle* is measured counterclockwise from the 0º line as shown in Figure 2. The symbol *r* is used for the radius and θ for the angle. A number in polar coordinates is written r∠θ. So the two points described in the last paragraph could also be written as (5.83, ∠59.0°) and (4.5, ∠243.4°). Unlike geographic maps, 0º is always to the *right* along the X-axis and 90°at the top along the Y-axis!

Angles are specified counterclockwise from the 0º axis. A negative value in front of an angle specifies an angle measured clockwise from the 0º axis. (Angles can also be given with reference to some arbitrary line, as well.) For example, –270º is equivalent to 90º, –90º is equivalent to 270º, 0º and –360º are equivalent, and +180º and –180º are equivalent. With an angle measured clockwise from the 0º axis, the polar coordinates of the second point in Figure 2 would be (4.5, ∠–116.6°).

In some calculations the angle will be specified in *radians* (1 radian = 360 / 2π degrees = 57.3°), but you may assume that all angles are in degrees in this book. Radians are used when *angular frequency* (ω) is used instead of frequency (*f*). Angular frequency is measured in units of 2π radians/sec and so ω = 2πf.

In electronics, it's common to use both the rectangular and polar-coordinate systems when dealing with impedance problems. The examples in the next few sections of this book should help you become familiar with these coordinate systems and the techniques for changing between them.

Complex Numbers

Mathematical equations that describe phases and angles of electrical quantities use the symbol *j* to represent the square root of minus one ($\sqrt{-1}$). (Mathematicians use *i* for the same purpose, but *i* is used to represent current in electronics.) The number *j* and any real number multiplied by *j* are called *imaginary numbers* because no real number is the square root of minus one. For example, 2*j*, 0.1*j*, 7*j*/4, and 457.6*j* are all imaginary numbers. Imaginary numbers are used in electronics to represent reactance, such as *j*13 Ω of inductive reactance or –*j*25 Ω of capacitive reactance.

Real and imaginary numbers can be combined by using addition or subtraction. Adding a real number to an imaginary number creates a hybrid called a *complex number*, such as 1 + *j* or 6 – 7*j*. These numbers come in very handy in radio, describing impedances, relationships between voltage and current, and many other phenomena.

If the complex number is broken up into its real and imaginary parts, those two numbers can also be used as coordinates on a graph using *complex coordinates*. This is a special type of rectangular-coordinate graph that is also referred to as the *complex plane*. By convention, the X axis coordinates represent the real number portion of the complex number and Y axis coordinates the imaginary portion. For example, the complex number 6 – 7*j* would have the same location as the point (6,–7) on a rectangular-coordinate graph. **Figure 3** shows the same points as Figure 1.

The number *j* has a number of interesting properties:

$1/j = -j$

$j^2 = -1$

$j^3 = -j$

$j^4 = 1$

Multiplication by *j* can also represent a phase shift or rotation of +90°. In polar coordinates *j* = 1 ∠90°.

WORKING WITH COMPLEX NUMBERS

Complex numbers representing electrical quantities can be expressed in either rectangular form (a + *j*b) or polar form (r∠θ) as described above. Adding complex numbers is easiest in rectangular form:

$$(a + jb) + (c + jd) = (a+c) + j(b+d) \qquad (9)$$

Multiplying and dividing complex numbers is easiest in polar form:

$$a\angle\theta_1 \times b\angle\theta_2 = (a \times b)\angle(\theta_1 + \theta_2) \qquad (10)$$

and

$$\frac{a\angle\theta_1}{b\angle\theta_2} = \left(\frac{a}{b}\right)\angle(\theta_1 - \theta_2) \qquad (11)$$

Converting from one form to another is useful in some kinds of calculations. For example, to calculate the value of two complex impedances in parallel you use the formula

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

To calculate the numerator (Z_1Z_2) you would write the impedances in polar form. To calculate the numerator ($Z_1 + Z_2$) you would write the impedances in rectangular form. So you need to be able to convert back and forth from one form to the other. Here is the procedure:

To convert from rectangular (a + *j*b) to polar form (r ∠ θ):

$$r = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1}(b / a) \qquad (12)$$

To convert from polar to rectangular form:

$$a = r \cos\theta \quad \text{and} \quad b = r \sin\theta \qquad (13)$$

Many calculators have polar-rectangular conversion functions built-in and they are worth learning how to use. Be sure that your calculator is set to the correct units for angles, radians or degrees.

Example: Convert 3 ∠ 60° to rectangular form:

$$a = 3 \cos 60° = 3\ (0.5) = 1.5$$

$$b = 3 \sin 60° = 3\ (0.866) = 2.6$$

$$3\angle 60° = 1.5 + j2.6$$

Example: Convert 0.8 + *j*0.6 to polar form:

$$r = \sqrt{0.8^2 + 0.6^2} = 1$$

$$\theta = \tan^{-1}(0.6 / 0.8) = 36.8°$$

$$0.8 + j0.6 = 1\angle 36.8°$$

Accuracy and Significant Figures

The calculations you have encountered in this chapter and will find throughout this handbook follow the rules for accuracy of calculations. Accuracy is represented by the number of *significant digits* in a number. That is, the number of digits that carry numeric value information beyond order of magnitude. For example, the numbers 0.123, 1.23, 12.3, 123, and 1230 all have three significant digits.

The result of a calculation can only be as accurate as its *least* accurate measurement or known value. This is important because it is rare for measurements to be more accurate than a few percent. This limits the number of useful significant digits to two or three. Here's another example; what is the current through a 12-Ω resistor if 4.6 V is applied? Ohm's Law says I in amperes = 4.6 / 12 = 0.3833333... but because our most accurate numeric information only has two significant digits (12 and 4.6), strictly applying the significant digits calculation rule limits our answer to 0.38. One extra digit is often included, 0.383 in this case, to act as a guide in rounding off the answer.

Quite often a calculator is used, and the result of a calculation fills the numeric window. Just because the calculator shows 9 digits after the decimal point, this does not mean that is a more correct or even useful answer.

THERMISTOR-BASED TEMPERATURE CONTROLLER

For many electronics applications, the exact value of temperature is not as important as the ability to maintain that temperature. The following project shows how to use a thermistor to create a circuit that acts as a temperature-controlled switch. The circuit is then used to detect over-temperature conditions and to act as a simple on/off temperature controller. For a much more detailed discussion of this material, see “Thermistors in Homebrew Projects,” by Bill Sabin, WØIYH, which is also included on the *Handbook* CD-ROM.

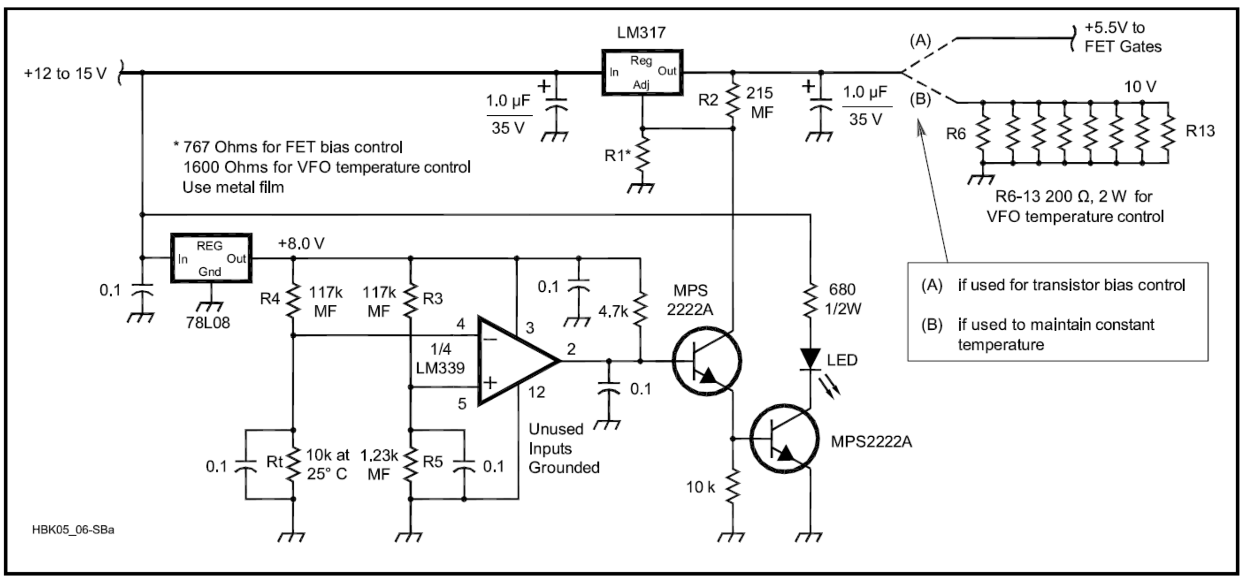

Figure 1 — Temperature controller for a power transistor (option A) or VFO (option B).

The circuit of **Figure 1** uses a thermistor (R1) to detect a case temperature of about 93 °C, with an on/off operating range of about 0.3 °C. A red LED warns of an over-temperature condition that requires attention. The LM339 comparator toggles when R_t = R5. Resistors R3, R4, R5 and the thermistor form a Wheatstone bridge. R3 and R4 are chosen to make the inputs to the LM339 about 0.5 V at the desired temperature. This greatly reduces self-heating of the thermistor, which could cause a substantial error in the circuit behavior. The RadioShack Precision Thermistor (RS part number 271-110) used in this circuit is rated at 10 kΩ ± 1% at 25 °C. It comes with a calibration chart from –50 °C to +110 °C that you can use to get an approximate resistance at any temperature in that range. For many temperature protection applications you don't have to know the exact temperature. You won't need an exact calibration, but you can verify that the thermistor is working properly by measuring its resistance at 20 °C (68 °F) and in boiling water (≅100 °C).

MOSFET POWER TRANSISTOR PROTECTOR

It is very desirable to compensate the temperature sensitivity of power transistors. In bipolar (BJT) transistors, thermal runaway occurs because the dc current gain increases as the transistors get hotter. In MOSFET transistors, thermal runaway is less likely to occur but with excessive drain dissipation or inadequate cooling, the junction temperature may increase until its maximum allowable value is exceeded. Suppose you want to protect a power amplifier that uses a pair of MRF150 MOSFETs.

You can place a thermistor on the heat sink close to the transistors so that the bias adjustment tracks the flange temperature. Another good idea is to attach a thermistor to the ceramic case of the FET with a small drop of epoxy. That way it will respond more quickly to a sudden temperature increase, possibly saving the transistor from destruction. Use the following procedure to get the desired temperature control:

The MRF150 MOSFET has a maximum allowed junction temperature of 200 °C. The thermal resistance θ_{JC} from junction to case is 0.6 °C per watt. The maximum expected dissipation of the FET in normal operation is 110 W. Select a case temperature of 93 °C. This makes the junction temperature 93 °C + (0.6 °C/W) × (110 W) =159 °C, which is a safe 41 °C below the maximum allowed temperature.

The FET has a rating of 300 W maximum dissipation at a case temperature of 25 °C, derated at 1.71 W per °C. At 93 °C case temperature, the maximum allowed dissipation is 300 W – 1.71 W/°C × (93 °C – 25 °C) = 184 W. The safety margin at that temperature is 184 – 110 = 74 W.

A very simple way to determine the correct value of R5 is to put the thermistor in 93 °C water (let it stabilize) and adjust R5 so that the circuit toggles. At 93°C, the measured value of the thermistor should be about 1230 Ω.

To use the circuit of Fig 1 to control the FET bias voltage, set R1 to be 767 Ω. This value, along with R2, adjusts the LM317 voltage regulator output to 5.5 V for the FET gate bias. When the thermistor heats to the set point, the LM339 comparator toggles on. This brings the FET gate voltage to a low level and completely turns off the FETs until the temperature drops about 0.3°C. Use metal film resistors throughout the circuit.

TEMPERATURE CONTROL FOR VFOS

The same circuit can be used to control the temperature of a VFO (variable frequency oscillator) with a few simple changes. Select R1 to be 1600 Ω (metal film) to adjust the LM317 for 10 V. Add resistors R6 through R13 as a heater element. Place the VFO in a thermally insulated enclosure. The eight 200-Ω, 2-W, metal-oxide resistors at the output of the LM317 supply about 4 W to maintain a temperature of about 33 °C inside the enclosure. The resistors are placed so that heat is distributed uniformly. Five are placed near the bottom and three near the top. The thermistor is mounted in the center of the box, close to the tuned circuit and in physical contact with the oscillator ground-plane surface, using a small drop of epoxy.

Use a massive and well-insulated enclosure to slow the rate of temperature change. In one project, over a 0.1 °C range, the frequency at 5.0 MHz varied up and down ±20 Hz or less, with a period of about five minutes. Superimposed is a very slow drift of average frequency that is due to settling down of components, including possibly the thermistor. These gradual changes became negligible after a few days of “burn-in.” One problem that is virtually eliminated by the constant temperature is a small but significant “retrace” effect of the cores and capacitors, and perhaps also the thermistor, where a substantial temperature transient of some kind may take from minutes to hours to recover the previous L and C values.

Thermistors in Homebrew Projects

Thermistors are inexpensive, readily available components. Better yet, they can greatly enhance the performance of your projects. Learn how to put them to work for you.

By William E. Sabin, W0IYH

Thermistors are interesting compnents that Amateurs can use to enhance their projects. Variations in circuit temperature that affect gain, distortion and control functions such as receiver AGC or transmitter ALC can be compensated. Dangers of self-destruction of overheated power transistors can be greatly reduced. Oscillator drift can be greatly reduced. This article discusses thermistor properties and shows three examples of how they can improve project performance. We will see first that some easy mathematics improves the understanding.

Mathematics of Thermistors

A thermistor is a small bit of intrinsic (undoped) semiconductor material between two wire leads. As temperature increases, the number of liberated hole-electron pairs increases exponentially, causing the resistance to decrease exponentially. This exponential nature is seen in the resistance equation:

$$R(T) = R(T0) \bullet e^{-\beta\left(\frac{1}{T0}-\frac{1}{T}\right)} \qquad \text{(Eq 1)}$$

where T is some temperature in Kelvins and $T0$ is a reference temperature, usually 298 K (25°C), at which the manufacturer specifies $R(T0)$. The constant β is experimentally determined by measuring resistance at various temperatures and finding the value of β that best agrees with the measurements. A simple way to get an approximate value of β (this is usually all we need in ham-gear design) is to make two measurements, at room temperature, say $T = 25\ °C$ (298 K) and $T0 \approx 100°C$ (373 K) in boiling water. Suppose the resistances are 10 kΩ and 938 Ω. Eq 1 is solved for β:

$$\beta = \frac{\ln\left[\frac{R(T)}{R(T0)}\right]}{\frac{1}{T}-\frac{1}{T0}} = \frac{\ln\left(\frac{938}{10000}\right)}{\frac{1}{373}-\frac{1}{298}} \approx 3507 \qquad \text{(Eq 2)}$$

Usually, the exact value of temperature is not as important as the ability to maintain that temperature. A better estimate of β, if needed, can by achieved by a linear regression method using the program *THERMIST.BAS*, downloadable from the *ARRL QEX* Web site.[1] This program takes the logarithm of both sides of Eq 1, which provides a linear relationship between $\log(R(T))$

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[1]You can download this package from the ARRL Web http://www.arrl.org/qexfiles/. Look for THERMIST.ZIP.

and (1/*T0* – 1/*T*). Five equally spaced data points are input to get the slope of the line, which is β.

The following examples illustrate a few of the main ideas of thermistor-circuit design that can be employed in a number of similar situations.

MOSFET Power-Transistor Protector

It is common practice to compensate the temperature sensitivity of power transistors. In bipolar (BJT) transistors, thermal runaway occurs because the dc current gain increases as the transistor gets hotter. The runaway condition is less likely to occur in MOSFET transistors, but with excessive drain dissipation or inadequate cooling the junction temperature will increase until its maximum allowable value is exceeded. A common procedure is to mount a diode or thermistor on the heat sink close to the transistors, so that the bias adjustment tracks the flange temperature. References 1 and 2 give detailed discussions of this.

A special problem occurs when a sudden large increase in transistor dissipation occurs. The flange temperature changes rather slowly because of the thermal capacitance (heat storage) of the heat sink. However, the junction temperature rises much more rapidly and can rise above the maximum limit before the correction circuit has a chance to function. If a thermistor is attached to the ceramic case with a small drop of epoxy as shown in Fig 1 it will respond much more rapidly (shorter time constant) and *may* (not guaranteed) save the transistor. The circuit (Reference 3) of Fig 2 detects a case temperature of about 93°C and completely turns off the FETs until the case temperature drops about 0.3°C, at which time the FETs are turned on again. A red LED on the front panel warns of an over-temperature condition that requires attention.

In Fig 2, a 4.7 V Zener (D1), *R1* (metal film) and *R*th (thermistor) are a voltage divider with an output of 0.6 V.

If this voltage decreases slightly (because the resistance of *R*th decreases slightly) Q1 starts to come out of saturation, Q2 quickly goes into saturation and the gate voltage of the FET goes to a low value, turning it off. At the same time, the 20-mA LED (RS 276-307) lights. The voltage divider equation is:

$$0.6 = 4.7 \bullet \left(\frac{R\text{th}}{R1 + R\text{th}} \right) \quad \text{(Eq 3)}$$

If we solve this for *R1*, we get:

$$R1 = 6.831 \bullet R\text{th} \quad \text{(Eq 4)}$$

If the value of *R*th is known at some temperature, then *R1* is the value that activates the circuit at that temperature. An interesting feature is that the voltage across the thermistor never exceeds about 0.6 V, and this greatly reduces the self-heating of the thermistor, which could otherwise cause a substantial error in the circuit behavior. The temperature variations of Q1 and D1 are small sources of error, so this simple circuit should be placed in a cool location, not directly on the heat sink.

The RadioShack precision thermistor (RS 271-110) is rated at 10 kΩ ±1% at 25 °C. It comes with a calibration chart from –50°C to +110°C that can be used to get an approximate resistance at some temperature. We are most interested in FET case temperatures in the range of 70°C to 100°C. It is assumed that very close temperature knowledge is not needed, but in order to be sure that the thermistor is okay, I measured its resistance at 20°C (68°F) and in boiling water (≈100°C). The circuit of

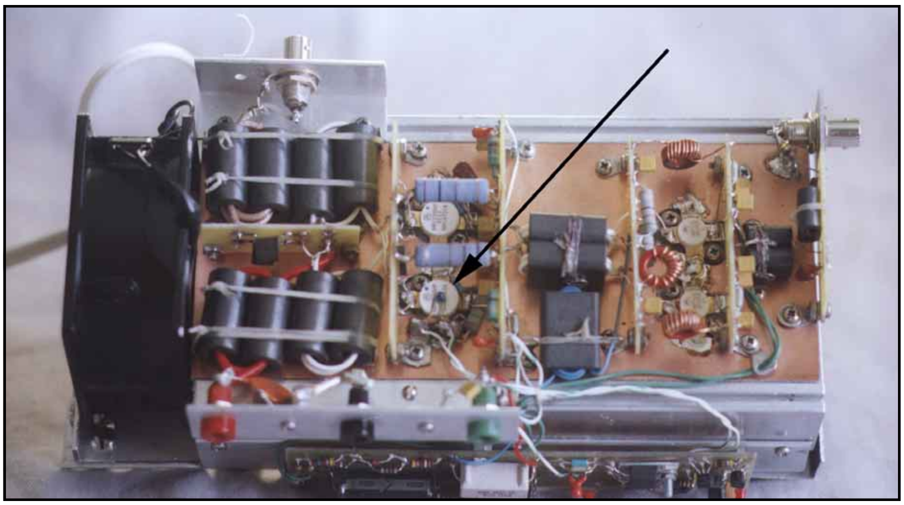

Fig 1—The thermistor attached to power MOSFET with a drop of epoxy.

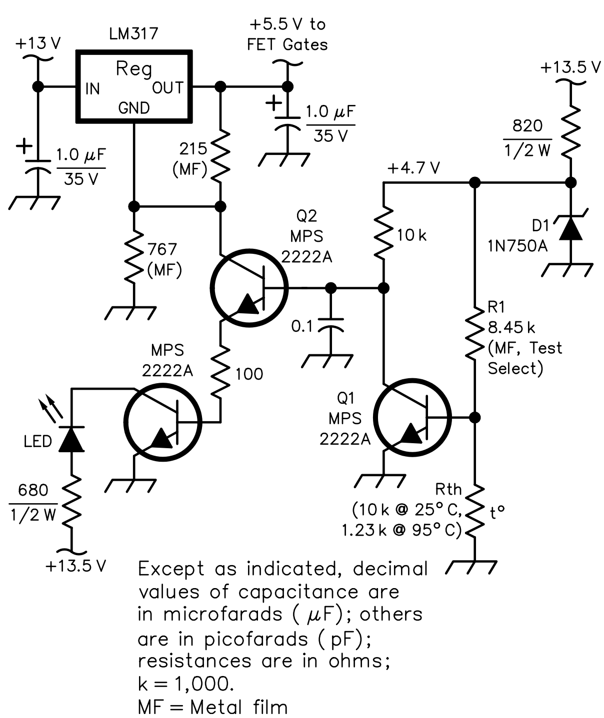

Fig 2—Schematic of a MOSFET temperature-protection circuit.

Fig 2 is intended for 50°C or greater when the 10-kΩ thermistor is used. At lower temperatures, the circuit of Fig 3 would be better.

The following procedure was used to get the desired temperature control:

1. The MRF150 MOSFET has a maximum allowed junction temperature of 200°C. The thermal resistance θ_{JC} from junction to case is 0.6°C/W.
2. The maximum expected dissipation of the FET in normal operation is 110 W.
3. I selected a case temperature of 93°C. This makes the junction temperature 93 + (0.6) (110) = 159°C, which is a safe 41°C below max.

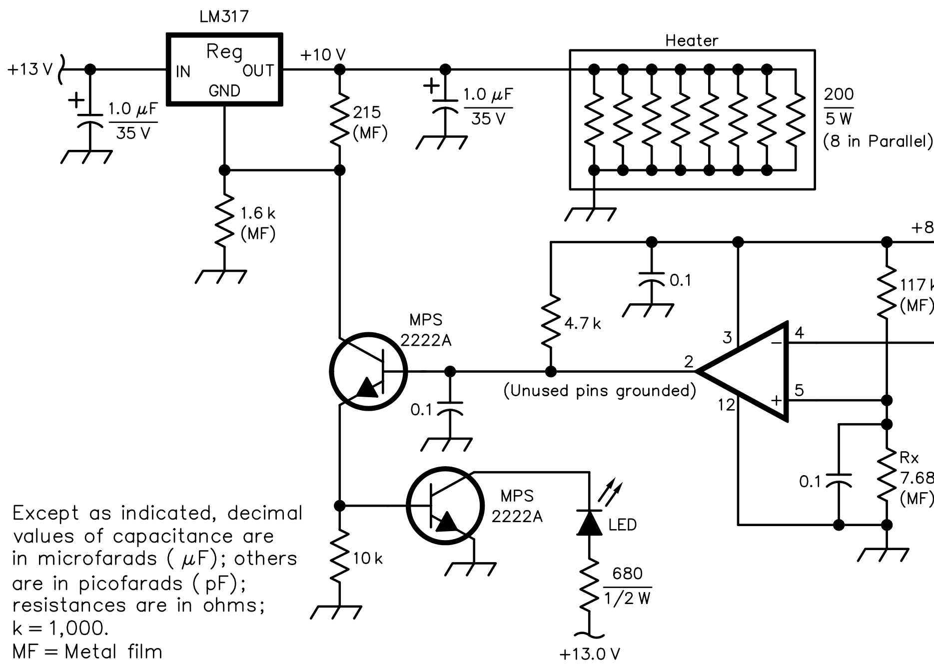

Fig 3—A VFO temperature-controller schematic. The eight 200-Ω resistors labeled "Heater" serve to warm the VFO enclosure

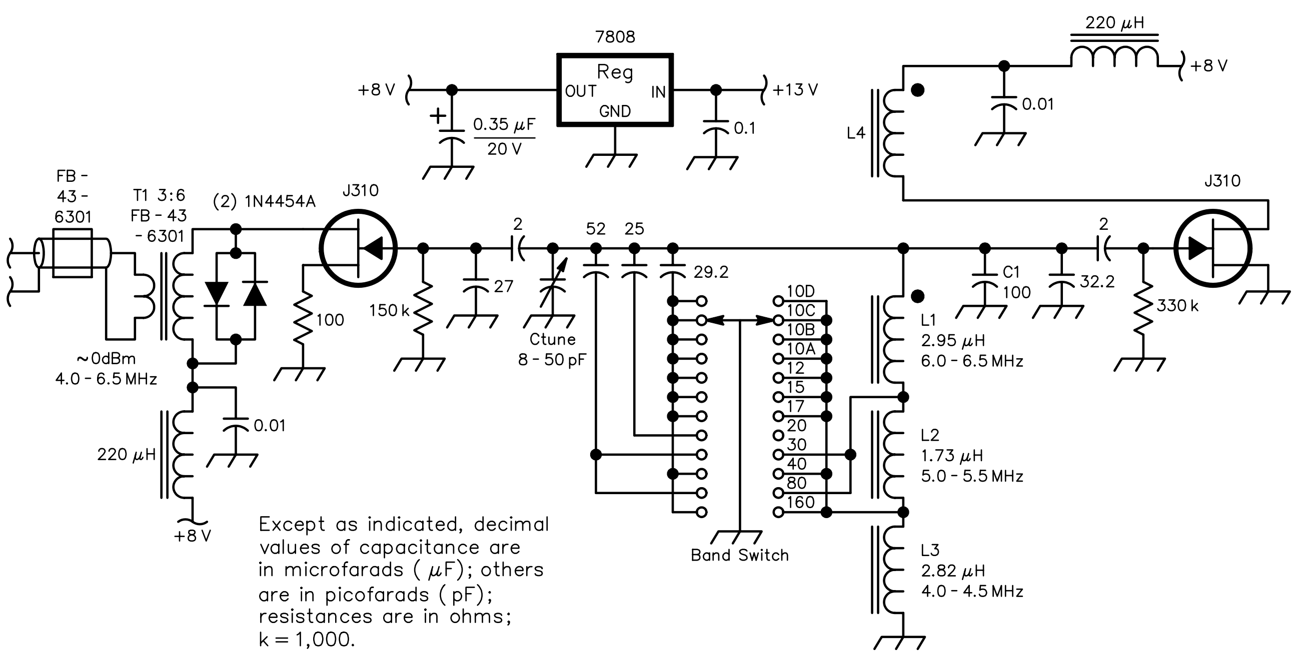

Fig 4—Schematic of a three-band VFO.
L1—22 t #24 AWG, wide spaced on a T68-7 core
L2—17 t #24 AWG, wide spaced on a T68-7 core
L3—22 t #24 AWG, wide spaced on a T68-7 core
L4—2 t #24 AWG link, close wound on L1 core
SW1—Electroswitch D4C0312N
Ctune—Hammarlund RMC-50-S

4. The FET has a rating of 300 W maximum dissipation at a case temperature of 25°C, derated at 1.71 W/°C. At 93°C case temperature, the maximum allowed dissipation is 300 – 1.71 (93 – 25) = 184 W. The safety margin at that temperature is 184 – 110 = 74 W.
5. A very simple way to determine the correct value of *R1* is to put the thermistor in 93°C water (let it stabilize) and adjust *R1* so that the circuit toggles. I found that 8450 Ω was the nearest standard value for a metal-film resistor. At 93°C, the measured value of the thermistor was about 1230 Ω.

Measurements of the circuit sensitivity determine the temperature values at which the circuit toggles on and off. I replaced the thermistor with a resistor decade box and measured resistances of 1224 Ω and 1234 Ω. Solve Eq 1 for the *T* that corresponds to each value of *R*:

$$T = \frac{\beta}{\ln\left(\frac{R}{R0}\right) + \frac{\beta}{T0}} \qquad \text{(Eq 5)}$$

which is easy to perform with a hand-held calculator or math program such as *MathCAD*. Using the two values of *R*, I found a temperature range of about 0.3°C.

A Temperature Controlled VFO

The circuit of Fig 3 is used to control the temperature of the three-band VFO shown in Fig 4, inside a thermally insulated enclosure. The Wheatstone bridge circuit with an LM339 comparator as a null detector is more sensitive and less temperature dependent than Fig 2. The ’339 works quite well at a level of 0.5 V at each of its two inputs. This circuit is preferable at lower temperatures, such as 30 to 35°C, where the thermistor resistance is in the 8 k to 7 kΩ range.

I use eight 200 Ω, 5 W metal-oxide resistors at the output of the LM317. The total heat applied is 4 W to maintain 33°C, and the resistors are placed so that their heat is distributed uniformly: Half are placed near the bottom and half near the top. The thermistor is mounted in the center of the box (see Fig 5), close to the tuned circuit and in physical contact with the oscillator ground-plane surface, using a small drop of epoxy.

The enclosure is homemade from sheet aluminum and angle stock. It is large enough that it has almost no effect on frequency. The outside is lined with 1/4-inch plexiglass and the inside surfaces are lined with 1/4-inch Styrofoam sheets. The tuning and bandswitch shafts are thermally insulated from the outside world, using plastic shaft couplers. Plexiglass blocks attach the box to the front panel. Electrical grounding is via an RF choke (for dc) and several 0.01 µF capacitors (for RF).

The temperature at the thermistor location is maintained within 0.1°C, as determined by thermistor resistance measurements. Using the method of the previous example to get the temperature range and knowing the frequency drift versus temperature coefficient of the VFO in parts per million (PPM), the frequency change can be found as follows:

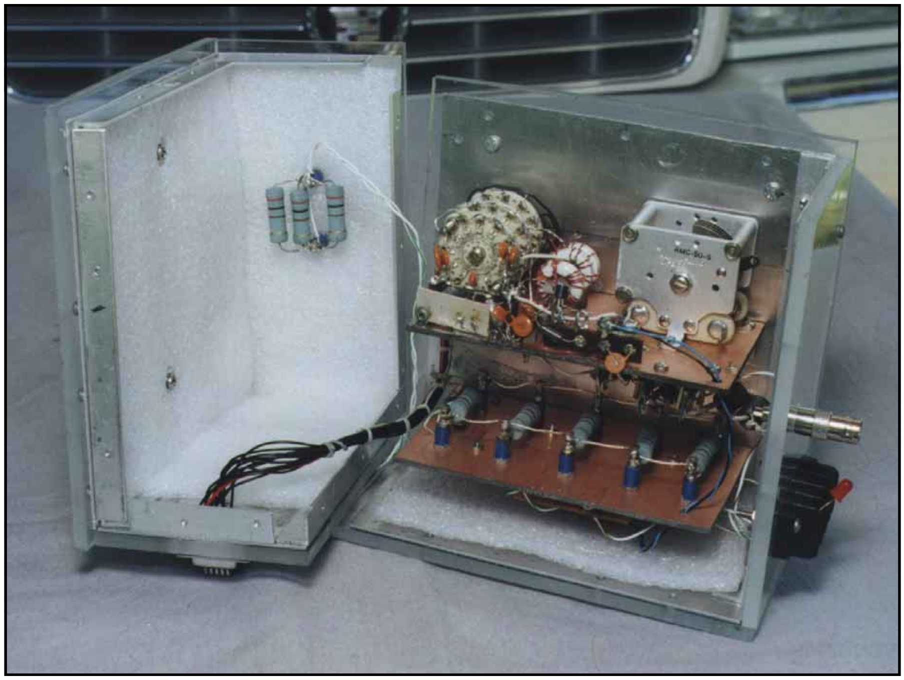

Fig 5—The three-band VFO with temperature control. Five of the eight 5-W resistors are mounted on a circuit board to the right of and slightly below center. The other three 5-W resistors are wired together slightly to the left and above center. The thermistor is not visible in this photo.

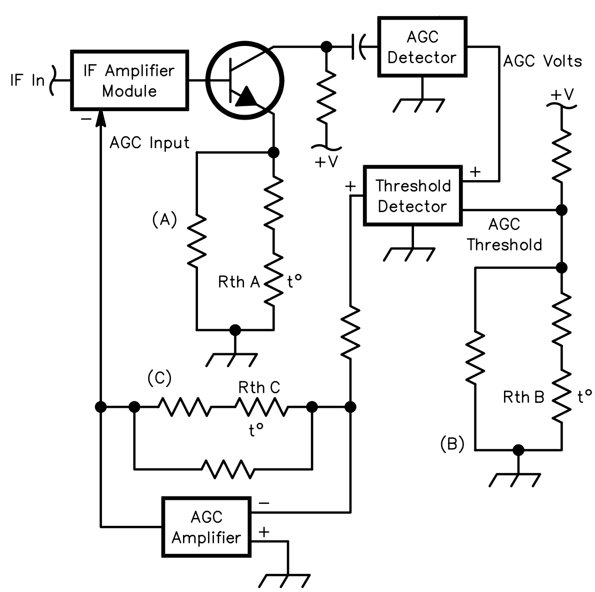

Fig 6—Temperature compensation of IF-amplifier and AGC circuitry: (A) IF amplifier, (B) AGC threshold, (C) AGC gain. Each compensation circuit uses two resistors and a thermistor (RthA, RthB or RthC).

$$\Delta F = F_{VFO} \bullet PPM \bullet \Delta T \quad \text{(Eq 6)}$$

Plugging in some numbers encountered for a 4.0 to 6.5 MHz VFO with PPM = –500/°C and Delta T = 0.1, before temperature compensation,

$$\Delta F = 6{,}000{,}000 \bullet \frac{-500}{1{,}000{,}000} \bullet 0.1 = -300 \text{ Hz} \quad \text{(Eq 7)}$$

which indicates that some improvement is needed, in particular the temperature coefficient. If the enclosure is massive or well insulated, the rate of temperature change can be slowed down. When a small temperature range of 0.1°C was maintained, the VFO temperature coefficient was improved by a factor of about seven with a negative-temperature-coefficient capacitor, C1. Inexpensive polystyrene capacitors, for example Mallory type SX, have a well-controlled negative (–120 PPM/°C) temperature coefficient that is intended to offset the positive temperature coefficient of inductors. (Caution: To prevent damage, use pliers on the leads as a heat sink when soldering.) No other temperature compensation was needed (probably fortuitously).

Temperature-compensating capacitors are available from Surplus Sales of Nebraska (**www.surplussales.com**). Other suitable fixed ceramic capacitors are Vishay 561 series, type 10TC NP0, which I have found are excellent. No variable trimmer caps are needed because each VFO band has a 25 kHz margin at each end and a calibrated analog dial is not used. The two-turn feedback winding in the J310 drain satisfies all three coils. The three inductors use Carbonyl-TH, T68-7 cores (white paint), which are claimed by Amidon to be the most temperature-stable mixture at normal room temperatures.

The oscillator finally turned out to be very slightly overcompensated. Over the 0.1°C range, the frequency varies ±20 Hz or less, with a period of about five minutes. Superimposed is a very slow drift of average frequency that is due to settling of component values, including possibly that of the RadioShack thermistor. These gradual changes became negligible after a few days of continuous operation.

One problem that is virtually eliminated by maintaining a constant temperature is the "retrace" effect on cores and capacitors. Because of "retrace," components subject to a substantial temperature transient of some kind may take several hours to recover their previous L and C values. The thermistor may also show a retrace effect.

Reference 4 shows ways to perform the temperature compensation operation and gives further references. An especially good method is to toggle the value of *Rx* (Fig 3) slightly so that a variation of ±0.5°C is created inside the VFO enclosure, and then do the temperature compensation. Because of the small average power dissipation in the VFO plus controller, it is economical to let the VFO run continuously so that initial warm-up drift (measured less than 1 kHz) and retrace are avoided.

If the VFO is mixed with crystal frequencies and then bandpass filtered (the "mix-master" approach) the final local oscillator (LO) can be quite stable and very clean spectrally. This is especially so if the crystals (±20 PPM/°C *maximum*) are temperature compensated, temperature controlled or even phase-locked to a reference (see Reference 5). An LO frequency counter (see Reference 6) offset by the IF is a very simple and excellent way to read the actual RF signal frequency to within ±50 Hz, if the reference crystal is of high quality and perodically adjusted to WWV. The frequency stability is quite adequate for HF SSB/CW, which are the primary applications for this equipment. Listening tests confirm that in SSB speech, slow frequency changes of ±50 Hz are hardly noticed. The reason for the three bands of the VFO (4.0-4.5, 5.0-5.5, 6.0-6.5 MHz) is to minimize spurious mixer products due to harmonic intermodulation that can slip through the LO bandpass filters.

Gain and AGC control

Thermistors are used to stabilize, or to vary in a controlled manner, the gain of an amplifier or an on-off threshold. Fig 6 sketches three examples. Fig 6A is the final IF amplifier stage that is part of an IF amplifier module. The thermistor compensates the gain variation of the module. Figs 6B and 6C are part of the receiver AGC circuit. The thermistors compensate for variations in AGC slope (dB/V) and vary the AGC threshold.

In all cases, use a resistor decade box to determine the resistance-temperature correlation and plot an RT curve. Then look for a resistor-thermistor circuit that seems reasonable. A *MathCAD* or *Excel* worksheet is then an elegant way to get the component values experimentally by comparing the thermistor-circuit temperature curve with the desired RT curve. Usually, a close approximation is good enough, and perfection is not justified. (Catalogs offer a wide assortment of thermistors for these projects.) The method suggested in Fig 6, one thermistor and two resistors, is a simple combination that gives a good approximation to the desired RT curve. In many cases, one of the resistors can be deleted. Reference 7 shows other useful and interesting thermistor applications.

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4. *ARRL Handbook* (Newington: ARRL) 1995 to 2000 editions, Chapter 14. Order No. 1832, $32. ARRL publications are available from your local ARRL dealer or directly from the ARRL. Check out the full ARRL publications line at **http://www.arrl.org/catalog/**.
5. See Chapter 17 of the *ARRL Handbook* for a description of the Ten-Tec Omni VI Plus transceiver.
6. Radio Adventures Co, RR4 Box 240, Summit Dr, Franklin, PA 16323; tel 814-437-5355, fax 814-437-5432; **information@radioadv.com**; **http://www.radioadv.com**. Model BK-172. For a description, see the 1999/2000 *ARRL Handbook,* Chapter 26.
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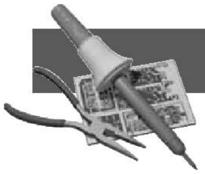

Experiment #32—Thevenin Equivalents

You can't buy a Thevenin Equivalent at the local electronics emporium, but it exists behind every pair of terminals—a fairly simple concept that you can use in many circumstances, a part of every electronic designer's tool set.

Terms to Learn

- *Duality*—the symmetry of opposite quantities, such as voltage and current or resistance and conductance, that allow electrical circuits to be described in terms of either.
- *Equivalent*—a circuit that behaves exactly the same as the original

Introduction

You've already encountered one aspect of Thevenin Equivalents by measuring the output impedance of the common-base amplifier circuit in Experiment 28. (All previous experiments are available at **www.arrl.org/tis/info/HTML/Hands-On-Radio**.) The output of the amplifier was acting like a perfect signal source in series with the measured resistance. Output impedance is just one instance of the more general idea of source impedance.

There are two kinds of sources that supply electrical energy in the form of voltage or current. An *ideal source* is one that can supply any amount of energy required while maintaining its rated voltage or current. For example, the ideal voltage source, shown in Figure 1, can satisfy Ohm's Law (I = V/R) through the attached resistor, whether the resistor is 1 MΩ or 1 µΩ.

An ideal source would be a handy thing; it doesn't really exist but needs only to be approximated. If I am trying to build an op-amp filter, my voltage source (that is, the power supply) may only have to supply a few mA to act as close to ideal as I need. To build a power grid, I need the Grand Coulee Dam's generators instead!

An actual voltage source can only supply a limited amount of power and still have a constant voltage or current. If I draw current from a battery, with a light bulb, for example, in Figure 1, the output voltage will drop. This voltage drop is caused by the battery's internal or source impedance. I can model the battery as an actual voltage source—an ideal source (V_S) in series with the internal impedance (R_S). The voltage at the battery's terminals, V_L, is reduced as the output current flows through the internal resistance:

$$I = V_S / (R_S + R_L) \qquad \text{[Eq 1]}$$

$$V_L = V_S - I\,R_S = (V_S - V_S\,R_S)/(R_S + R_L)$$
$$= V_S\,[1 - R_S / (R_S + R_L)] \qquad \text{[Eq 2]}$$

Equation 1 shows that for an actual voltage source as R_L increases, the output voltage will drop and vice versa. If R_L is infinite (no load), the output or terminal voltage will be the same as that of the ideal source, since no current flows through the internal impedance. If R_L is zero (a short circuit), then the current is limited to V_S / R_S.

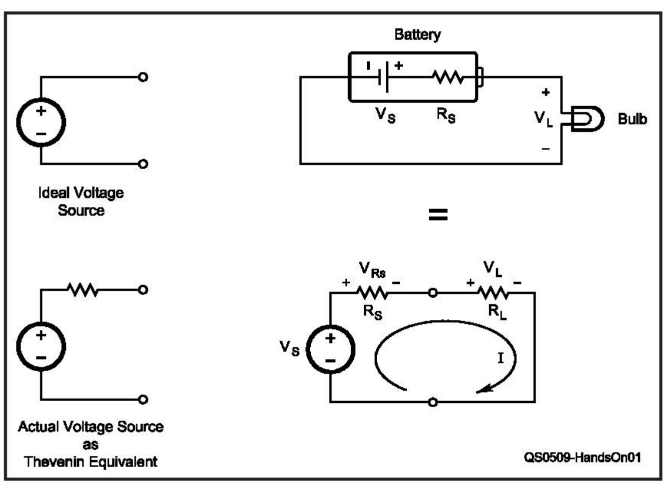

Figure 1—The Thevenin equivalent circuit is constructed from an ideal voltage source in series with a resistance.

Thevenin's Insight

From the Wikipedia (**www.wikipedia.com**): "Leon Charles Thevenin (1857-1926) was a French telegraph engineer who extended Ohm's law to the analysis of complex electrical circuits." Thevenin's main legacy is a simple but powerful statement: For a circuit made up of any combination of voltage sources and resistors, the behavior at a pair of terminals to that circuit can be completely replicated by a circuit consisting of a single ideal voltage source, called the Thevenin voltage (V_{TH}), in series with a single resistance, called the Thevenin resistance (R_{TH}). That is Thevenin's Theorem, and the source and resistance comprise the Thevenin Equivalent in Figure 1.

That's a pretty powerful idea! If the original circuit, no matter how complicated, was out of your sight with only a pair of terminals visible, you couldn't tell whether the circuit was the complicated one or its Thevenin equivalent. All you know about it is the behavior at its external terminals. Thevenin's Theorem also works for ac signals and circuits, using ac signal sources and impedances. In general, the theorem is true as long as all of the sources and components respond linearly to voltage and current—no diodes or relays, for example.

Equivalence, More or Less

When you're designing or analyzing a circuit, it's handy to be able to replace parts of it with simplified circuit bits that act the same but are easier to analyze. These are equivalent circuits. For example, if I'm working on an amplifier circuit, I need to know the characteristics of what is connected to its input terminals. I don't really want to deal with the whole circuit of the signal source, so I replace it with its Thevenin equivalent. The Thevenin resistance is what you measured as output impedance in Experiment #28.

An equivalent circuit doesn't need to be a Thevenin equivalent. If you can replace a circuit of resistors in parallel with a

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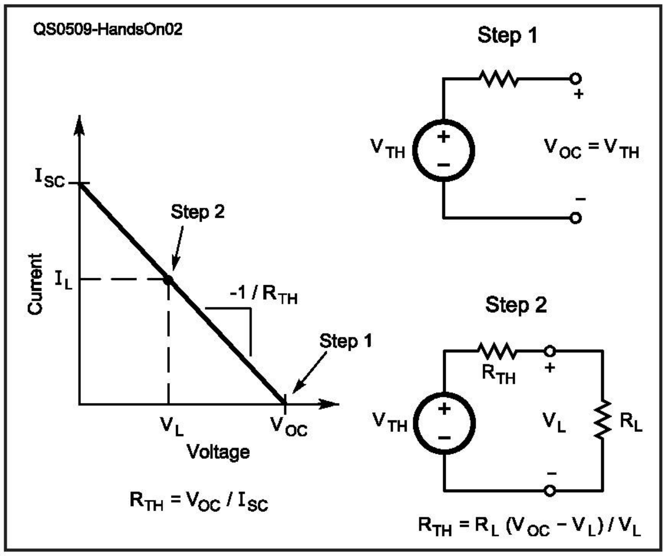

Figure 2—The values of the Thevenin equivalent circuit can be determined by simple measurements of voltage and current.

different circuit that has resistors in series, the new circuit is the original's series equivalent. (Going the other way, there's also a parallel equivalent.) A circuit that responds the same only at dc is the dc equivalent. All "equivalent circuit" means is that you can replace a circuit with its equivalent without affecting the external behavior.

Measuring a Thevenin Equivalent

Let's figure out the Thevenin equivalent circuit for a battery by determining V_{TH} and R_{TH}. A battery makes a pretty good voltage source, but it still has some internal resistance. How can you figure out what that internal resistance is? We'll use almost the same technique as was used to measure output impedance. Figure 2 illustrates the two steps and how the measurements allow you to determine both V_{TH} and R_{TH}.

- Obtain a 1.5 V AAA or 9 V battery. Do *not* use a larger battery because they can supply enough current to cause a burn or damage themselves.
- The load will be a 100 Ω, 1 W resistor; either buy one or make it from several lower-power resistors. Measure the exact value with your voltmeter. It doesn't have to be exactly 100 Ω; anything from 25 to 100 Ω will do. Don't use a light bulb; the resistance changes dramatically as it heats up.
- With no load connected at all, measure the open-circuit voltage of the battery. Since no current flows in R_{TH}, this is the same voltage as V_{TH}.
- Connect your voltmeter to the resistor leads, and using pliers or tweezers apply the resistor leads directly to the battery terminals without using clip leads or other connectors. Don't hold the resistor or its leads in your fingers, as it may get hot!
- The voltage will drop by some amount to V_L. For fresh batteries the voltage may not drop much. You can increase the voltage drop by lowering R_L, taking care to provide enough power dissipation. ($P = V^2 / R$)
- Use the equation in Figure 2 to determine R_{TH}. Congratulations: You just determined the Thevenin equivalent of your battery! Whatever is inside your battery—more batteries, chemicals, or a tiny hamster on a wheel—can be replaced by an ideal voltage source, V_{TH}, and series resistance, R_{TH}.
- Experiment with fresh and depleted batteries of the same type to see how their Thevenin circuit changes with energy level. Try different types of batteries, as well, taking care to avoid excessive heat dissipation in the load resistor.

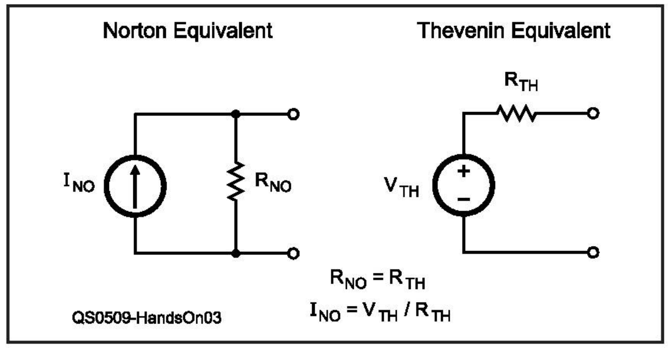

Figure 3—The Norton Equivalent and Thevenin Equivalent circuits exhibit exactly the same behavior at their outputs and can replace each other directly.

The graph in Figure 2 illustrates the process of determining V_{TH} and R_{TH}. With no current drawn from the source, the open-circuit and Thevenin voltages are equivalent. If you short-circuit the output terminals, the current is limited to I_{SC} by R_{TH}. The negative reciprocal of the slope of the line between V_{OC} and I_{SC} is equal to R_{TH}. It's often not advisable to use a short circuit because of the potential damage to the circuit being tested or the circuit tester. Use an intermediate point with a safe current and determine R_{TH} from the slope of the line between that point and the open-circuit point.

Norton Equivalent

There is another way to create an equivalent circuit using an ideal current source and a resistance in parallel; this is called a Norton Equivalent, shown in Figure 3. The ideal current source supplies a fixed amount of current to whatever load is connected to its terminals. Surprisingly, R_{TH} has the same value in both the Norton and Thevenin equivalent circuits!

If you short-circuit a Thevenin circuit, $I_{SC} = V_{TH} / R_{TH}$. If the output is open-circuited, the voltage across the terminals is $I_{SC} \times R_{TH} = V_{OC} = V_{TH}$. Construct the Norton Equivalent by replacing the Thevenin voltage source with a current source of value I_{SC} and place R_{TH} across it. You get exactly the same graph of current and voltage at the output terminals—the definition of an equivalent circuit.

Dual Challenges

Thevenin and Norton circuits are a window into the world of *duality* in electronics where electrical behavior can be defined in terms of current or voltage. Other dual quantities include resistance and conductance, impedance and admittance, series and parallel, and node and mesh (from Experiment #29). The use of one parameter or the other is just a matter of convenience and always leads to the same answer for electrical energy and power.

Shopping List

- AAA or 9 V battery
- 100 Ω, 1 W resistor

Suggested Reading

Thevenin and Norton equivalents are discussed with examples on pages 4.5 and 4.6 of *The 2005 ARRL Handbook*. A more extensive discussion is available on-line at **www.allaboutcircuits.com/vol_1/chpt_10/7.html**. That Web page also has extensive links to other aspects of electrical circuits.

Next Month

In October, we'll get to know the magnetic personality of one of the electronic world's most common components—the transformer. QST